A New Test for Passivity of Descriptor Systems

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Outline

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Linear Descriptor Systems

Linear time-invariant systems in generalized state-space form:

\[ E \dot{x}(t) = Ax(t) + Bu(t), \]
\[ y(t) = Cx(t) + Du(t), \]

arise, e.g., in

- control and simulation of coupled systems,
- control of multibody (mechanical) systems,
- manipulation of fluid flow (e.g., semi-discretized Navier-Stokes equations),
- circuit simulation, VLSI chip design, in particular modeling of interconnect via RLC networks,
- simulation of MEMS and NEMS (micro-/nano-electro-mechanical systems).
**Assumptions**

- $n$ generalized states / descriptor variables, i.e., $x(t) \in \mathbb{R}^n$;
- $m$ “inputs”, i.e., $u(t) \in \mathbb{R}^m$;
- $m$ “outputs”, i.e., $y(t) \in \mathbb{R}^m$;
- $A - \lambda E$ regular, i.e., $\exists \lambda \in \mathbb{C} : \det(A - \lambda E) \neq 0$;
- $A - \lambda E$ stable, i.e., $\lambda(A, E) \subset \mathbb{C}^- \cup \{\infty\} \Rightarrow$ system is stable.

Corresponding transfer function:

$$G(s) = C(sE - A)^{-1}B + D$$
Regular matrix pencils are equivalent \(([A, E] \mapsto (PAQ, PEQ))\) to their Weierstraß canonical form (WCF)

\[
\begin{bmatrix}
J & 0 \\
0 & I_{n_{\infty}}
\end{bmatrix} - \lambda
\begin{bmatrix}
I_{n_f} & 0 \\
0 & N
\end{bmatrix}.
\]

Here:

- \(J\) contains Jordan blocks corresponding to the \(n_f = n - n_{\infty}\) finite eigenvalues,
- \(N\) is nilpotent and contains Jordan blocks to the \(n_{\infty}\) infinite eigenvalues,
- \(\nu :=\) size of largest Jordan block in \(N\) is called the (algebraic) index of \(A - \lambda E\).
Matrix pencil equivalence implies restricted system equivalence (RSE):

\[(E, A, B, C, D) \sim (PEQ, PAQ, PB, CQ, D)\] for any nonsingular \(P, Q \in \mathbb{R}^{n \times n}\).

With the coordinate transformation \(x \mapsto Px =: \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\), and a corresponding partitioning

\[PB = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \quad CQ = \begin{bmatrix} C_1 & C_2 \end{bmatrix},\]

RSE and the WCF imply the decoupling

\[
\begin{aligned}
\dot{x}_1 &= Jx_1 + B_1u, \\
y_1 &= C_1x_1 + Du \\
N\dot{x}_2 &= x_2 + B_2u, \\
y_2 &= C_2x_2
\end{aligned}
\]

\{ slow subsystem \} \quad \{ fast subsystem \}
Additive Decomposition of Transfer Function

Partial information about a descriptor system is obtained from its block-diagonal form

\[ S(A - \lambda E)T = \begin{bmatrix} A_f & 0 \\ 0 & A_\infty \end{bmatrix} - \lambda \begin{bmatrix} E_f & 0 \\ 0 & E_\infty \end{bmatrix}, \quad SB = \begin{bmatrix} B_f \\ B_\infty \end{bmatrix}, \quad CT = \begin{bmatrix} C_f & C_\infty \end{bmatrix}, \]

where \( S, T \in \mathbb{R}^{n \times n} \) are nonsingular. This yields

**slow-fast decoupling:**

\[
\begin{align*}
E_f \dot{x}_f &= A_f x_f + B_f u, \\
y_f &= C_f x_f + D u \\
E_\infty \dot{x}_\infty &= A_\infty x_\infty + B_\infty u, \\
y_\infty &= C_\infty x_\infty
\end{align*}
\]

**additive decomposition of** \( G(s) \):

\[ G(s) = G_f(s) + G_\infty(s) = C_f (sE_f - A_f)^{-1} B_f + D + C_\infty (sE_\infty - A_\infty)^{-1} B_\infty. \]
Markov Parameters

A rational transfer function $G(s) = C(sE - A)^{-1}B + D$ has a power series expansion (Laurent series) at $s_0 = \infty$ of the form

$$G(s) = \sum_{k=-\infty}^{q} s^k M_k,$$

where $M_k \in \mathbb{R}^{m \times m}$ are the Markov parameters of $G$ and $q \leq \nu$.

This implies

$$G(s) = G_{sp}(s) + M_0 + sM_1 + \sum_{k=2}^{q} s^k M_k = G_p(s) + sM_1 + \sum_{k=2}^{q} s^k M_k,$$

where

- $G_{sp}(s)$ is strictly proper, i.e., $\lim_{s \to \infty} G_{sp}(s) = 0$;
- $G_p(s) := G_{sp} + M_0$ is proper, i.e., $\lim_{s \to \infty} G_p(s)$ is finite.
Passive Systems

**Definition:** A system is **passive** if \[ \int_{-\infty}^{t} u(\tau)^T y(\tau) \, d\tau \geq 0 \quad \forall t \in \mathbb{R}, \forall u \in L_2(\mathbb{R}, \mathbb{R}^m). \]

"The system cannot generate energy."

**Theorem:** system is passive \(\iff\) its transfer function is positive real

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**Definition:** A **real**, rational matrix-valued function \( G : \mathbb{C} \rightarrow \mathbb{C}^{m \times m} \) is (strictly) **positive real** if

1. \( G \) is analytic in \( \mathbb{C}^+ := \{ s \in \mathbb{C} \mid \text{Re}(s) > 0 \} \),
2. \( G(s) + G^T(s) \geq 0 \) for all \( s \in \mathbb{C}^+ \) (\( G(s) + G^T(s) > 0 \) for all \( s \in \mathbb{C}^+ \)).
Motivation

Guaranteeing passivity of descriptor systems is necessary in

Model reduction for passive systems:

- Task often encountered in circuit simulation, VLSI chip design.
- Padé-type methods in general do not preserve passivity, post-processing necessary [Bai/Feldmann/Freund ’98,’01].
  - SyPVL preserves passivity for RLC circuits [Feldmann/Freund ’96,’97].
  - LR-ADI/dominant subspace approximation can preserve passivity [Li/White ’01].
- But in general: passivity not guaranteed for reduced-order models computed by Padé-type methods, needs to be checked!

Model Reference Adaptive Control:

passivity of “input-to-tracking error” transfer function guarantees tracking property

[Landau 1979, Dai 1989, . . .].
Hybrid Methods for Model Reduction

Recent “trend” in model reduction of LARGE-scale systems (MEMS, CFD):
“LARGE” = \( n > 100,000 \).

Apply method for large sparse eigenproblems like
- Jacobi-Davidson,
- Krylov-subspace method (Lanczos, Arnoldi),

to compute projector onto low-dimensional \( (n \approx 1000) \) subspace using
- modal truncation,
- Padé(-type) approximation,

ideas, then reduce the intermediate model further by balancing-related techniques.

For passive systems, need to check that intermediate system is still passive.
The Positive Real Lemma

Kalman/Yakubovich/Popov/Anderson 1962–67

**Theorem:**
Let \((A, B, C, D)\) be a minimal realization of a linear time-invariant system with transfer function \(G(s)\).

a) \(G(s)\) is positive real \(\iff\) \(\exists\) a solution \(X \geq 0\) to the LMI

\[
\begin{bmatrix}
A^T X + X A & X B - C^T \\
B^T X - C & -(D + D^T)
\end{bmatrix} \leq 0.
\]

b) If \(D + D^T > 0\), then \(G(s)\) is strictly positive real \(\iff\) the algebraic Riccati equation (ARE)

\[
A^T X + X A + (X B - C^T)(D + D^T)^{-1}(B^T X - C) = 0,
\]

has a stabilizing solution \(X\).
A Positive Real Lemma for Descriptor Systems

FREUND/JARRE 2000/2004

Theorem:

a) (Sufficiency) Let \((E, A, B, C, D)\) be a realization of a linear descriptor system with transfer function \(G(s)\). If the LMIs

\[
\begin{bmatrix}
    A^T X + X^T A & X^T B - C^T \\
    B^T X - C & -(D + D^T)
\end{bmatrix} \leq 0, \quad E^T X = X^T E \geq 0
\]

have a solution \(X\), then \(G(s)\) is positive real.

b) (Necessity) Let \((E, A, B, C, D)\) be a minimal realization of a linear descriptor system with transfer function \(G(s)\) satisfying

\[
D + D^T \geq M_0 + M_0^T, \quad \text{where } M_0 \text{ is the 0th Markov parameter of } G.
\]

Then, if \(G(s)\) is positive real, there exists a solution of the LMIs given above.
Testing Positive Realness of Descriptor Systems

- Testing positive realness via LMIs often not feasible due to computational complexity ($O(n^6)$, employing structure $O(n^5)$).

- Even in case $D + D^T > 0$, the Riccati equation/Hamiltonian eigenproblem test is not applicable if $E$ is singular.

- Eigenvalue-based test for scalar transfer functions of standard and descriptor systems exists, but no generalization to MIMO is known. [Bai/Freund 2000]

- For standard systems, recursive reduction procedure can be applied: $(A, B, C, D) \rightarrow (\tilde{A}, \tilde{B}, \tilde{C}, \tilde{D})$ with $\tilde{D} + \tilde{D}^T$ nonsingular; $G(s)$ is then positive real if $\tilde{G}(s)$ is strictly positive real. [Weiss/Wang/Speyer 1994]

**Goal:** algebraic test for positive realness that only requires orthogonal coordinate transformations and has complexity $O(n^3)$. 
Markov Parameters and Positive Realness

Anderson/Vongpanitlerd 1973, Freund/Jarre 2000:

**Theorem:** Given a rational matrix-valued function

\[
G(s) = G_p(s) + sM_1 + \sum_{k=2}^{q} s^k M_k,
\]

then \( G(s) \) is positive real \( \iff \)

1. \( G_p(s) \) is positive real,
2. \( M_1 \geq 0 \),
3. \( M_k = 0, \ k = 2, 3, \ldots, q \).

**Remarks:**
Condition 3. is trivially satisfied if the index of \( A - \lambda E \) satisfies \( \nu \leq 2 \).

Conditions 2. + 3. are trivially satisfied if \( \nu \leq 1 \).
Method I

Algorithm:
1. Compute an additive decomposition of $G(s)$,

\[
G(s) = G_f(s) + G_\infty(s) \\
= [C_f(sE_f - A_f)^{-1}B_f + D] + [C_\infty(sE_\infty - A_\infty)^{-1}B_\infty].
\]

2. Test if $G_\infty(s) = -C_\infty A_\infty^{-1}B_\infty - sC_\infty A_\infty^{-1}E_\infty A_\infty^{-1}B_\infty$.
3. Test if $-C_\infty A_\infty^{-1}E_\infty A_\infty^{-1}B_\infty$ is positive semidefinite.
4. Test if $G_f(s) - C_\infty A_\infty^{-1}B_\infty$ is positive real.

Remarks:
- Step 1. may be ill-conditioned as non-orthogonal transformations are required.
- Step 2. could be tested using sufficiently many interpolation points $s_k$ so that $s_k E - A$ is well-conditioned.
- Method I is feasible for index-1 descriptor systems as Steps 2.+3. are redundant and Step 1. is “easier”.
Method II

Main ideas:

- assume minimality (otherwise, compute minimal realization first, using e.g., MATLAB Descriptor Systems Toolbox [Varga 2000–2005]).
- avoid additive decomposition, instead compute new reduced form of $A - \lambda E$ using only orthogonal equivalence transformations;
- from reduced form obtain explicit expressions for $G_p(s)$, $M_1$, and $M_k$ for $k = 2, \ldots, q$;
- the conditions $M_k = 0$ for $k \geq 2$ can be checked via the rank indices of the new reduced form;
- testing $M_1 \geq 0$ by adopted algorithm employing orthogonal decompositions only (complexity $O(n^3)$);
- testing $G_p(s)$ positive real using reduction to “strictly positive real” check.
Method II: Orthogonal Reducing Equivalence Transformation

Lemma:

For any regular pencil $A - \lambda E$ there exist orthogonal matrices $U, V \in \mathbb{R}^{n \times n}$ such that

$$U(A - \lambda E)V = \begin{bmatrix}
A_{11} - \lambda E_{11} & A_{12} - \lambda E_{12} & A_{13} - \lambda E_{13} & A_{14} - \lambda E_{14} \\
0 & A_{22} & A_{23} - \lambda E_{23} & A_{24} - \lambda E_{24} \\
0 & 0 & A_{33} & A_{34} \\
0 & 0 & 0 & A_{44}
\end{bmatrix},$$

where $\text{rank}(E_{11}) = n_1$, $\text{rank}(E_{23}) = n_3$, $\text{rank}(A_{44}) = n_4$, and

$$\text{rank} \left( \begin{bmatrix}
A_{22} & A_{23} - \lambda E_{23} \\
0 & A_{33}
\end{bmatrix} \right) = n_2 + n_3 \ \forall \lambda \in \mathbb{C}.$$

Proof: constructive algorithm using 3 URVs (SVDs), 1 RQ factorization, 1 generalized Schur decomposition $\sim$ complexity $O(n^3)$. 

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Method II: Reducing Equivalence Transformation

Note: this step is only needed for theoretical purposes, no computations erquired!

Due to partitioning of transformed matrix pencil, as an intermediate step to WCF we can find nonsingular matrices

\[ X = \begin{bmatrix} I_{n_1} & X_2 & X_3 & X_4 \\ 0 & I_{n_3} & 0 & 0 \\ 0 & 0 & I_{n_2} & 0 \\ 0 & 0 & 0 & I_{n_4} \end{bmatrix}, \quad Y = \begin{bmatrix} I_{n_1} & Y_2 & Y_3 & Y_4 \\ 0 & I_{n_2} & 0 & 0 \\ 0 & 0 & I_{n_3} & 0 \\ 0 & 0 & 0 & I_{n_4} \end{bmatrix}, \]

such that

\[
XU(A - \lambda E)VY = \begin{bmatrix} A_{11} - \lambda E_{11} & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} - \lambda E_{23} & 0 \\ 0 & A_{33} & A_{34} - \lambda E_{34} & 0 \\ 0 & 0 & A_{44} & A_{44} \end{bmatrix}.
\]
**Method II: Representation of** $G_p$ **and** $M_k$

After the RSE transformation implied by $XU, VY$ we get

$$G(s) = C_1(sE_{11} - A_{11})^{-1}(B_1 + X_2B_2 + X_3B_3 + X_4B_4) + D$$

$$+ \begin{bmatrix} C_2 + C_1Y_2 & C_3 + C_1Y_3 & C_4 + C_1Y_4 \end{bmatrix} \begin{bmatrix} A_{22} & -sE_{23} + A_{23} & -sE_{24} + A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} B_2 \\ B_3 \\ B_4 \end{bmatrix}.$$ 

$\implies$ formulae for $G_p, M_k$:

$$G_p(s) = C_1(sE_{11} - A_{11})^{-1}(B_1 + X_2B_2 + X_3B_3 + X_4B_4) + D$$

$$- \begin{bmatrix} C_2 + C_1Y_2 & C_3 + C_1Y_3 & C_4 + C_1Y_4 \end{bmatrix} \begin{bmatrix} A_{22} & A_{23} & A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} B_2 \\ B_3 \\ B_4 \end{bmatrix},$$

$$M_k = - \begin{bmatrix} C_2 + C_1Y_2 & C_3 + C_1Y_3 & C_4 + C_1Y_4 \end{bmatrix} \left( \begin{bmatrix} A_{22} & A_{23} & A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} 0 & E_{23} & E_{24} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right)^k \times \begin{bmatrix} A_{22} & A_{23} & A_{24} \\ 0 & A_{33} & A_{34} \\ 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} B_2 \\ B_3 \\ B_4 \end{bmatrix}, \quad k = 1, \ldots, n_2 + n_3.$$
Method II: Simplification of $G_p, M_k$

$E_{23}$ nonsingular $\implies$ for $k = 1, \cdots, n_2 + n_3$

$$
M_k = - \left[ \begin{array}{cc} C_2 + C_1 Y_2 & C_3 + C_1 Y_3 \end{array} \right] \left( \begin{array}{cc} A_{22} & A_{23} \\ 0 & A_{33} \end{array} \right)^{-1} \left( \begin{array}{cc} 0 & E_{23} \\ 0 & 0 \end{array} \right)^k \left( \begin{array}{cc} A_{22} & A_{23} \\ 0 & A_{33} \end{array} \right)^{-1} \times \\
\times \left( \begin{array}{c} B_2 \\ B_3 \end{array} \right) - \left[ \begin{array}{cc} A_{24} - A_{23} E_{23}^{-1} E_{24} \\ A_{34} - A_{33} E_{23}^{-1} E_{24} \end{array} \right] A_{44}^{-1} B_4.
$$

With this, the transfer function becomes

$$
G(s) = \hat{C} \left( s \begin{array}{cc} E_{11} \\ 0 & 0 \\ 0 & 0 \end{array} \right) - \left[ \begin{array}{cc} A_{11} \\ 0 & A_{22} \\ 0 & A_{33} \end{array} \right]^{-1} \hat{B} + \hat{D},
$$

where with notation from WCF

$$
E_{11} \sim I_{n_f}, \quad A_{11} \sim J, \quad \begin{array}{cc} 0 & E_{23} \\ 0 & 0 \end{array} \sim N, \quad \begin{array}{cc} A_{22} & A_{23} \\ 0 & A_{33} \end{array} \sim I_{n_{\infty}}.
$$
**Method II: Testing** $M_k = 0$ for $k = 2, \ldots, q$

**Lemma:**

a) $n_2 = n_3 \implies M_k = 0$ for $k = 2, \ldots, n_2 + n_3$.

b) $G(s)$ positive real $\implies n_2 = n_3$.

**Proof:**

a) $n_2 = n_3 \implies \begin{bmatrix} A_{22} & A_{23} \\ 0 & A_{33} \end{bmatrix}^{-1} = \begin{bmatrix} A_{22}^{-1} & \ast \\ 0 & A_{33}^{-1} \end{bmatrix} \implies \left( \begin{bmatrix} A_{22} & A_{23} \\ 0 & A_{33} \end{bmatrix}^{-1} \begin{bmatrix} 0 & E_{23} \\ 0 & 0 \end{bmatrix} \right)^k = 0 \ \forall \ k \geq 2.

b) [Freund/Jarre]: $G$ minimal, positive real $\implies$ nilpotent matrix $N$ from WCF

- is either empty $\implies n_2 = n_3 = 0$;
- or has only $2 \times 2$ Jordan blocks $\implies n_2 + n_3$ even.

Now, $N \sim \begin{bmatrix} 0 & E_{23} \\ 0 & 0 \end{bmatrix} \sim \begin{bmatrix} 0 & I_{n_3} \\ 0 & 0 \end{bmatrix}$, with one “1” from each Jordan block $\implies n_2 = n_3.$
Method II: Case $n_3 = 0$

$A - \lambda E$ regular $\implies n_2 = n_3 = 0 \implies M_1 = 0$ and

$$G_p(s) = G(s) = \begin{bmatrix} C_1 & C_4 \end{bmatrix} \left( s \begin{bmatrix} E_{11} & E_{14} \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A_{11} & A_{14} \\ 0 & A_{44} \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1 \\ B_4 \end{bmatrix} + D.$$

$$= \begin{bmatrix} C_1 & C_2 \end{bmatrix} \left( s \begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + D,$$

with $E_{11}, A_{22}$ nonsingular.

Thus, from transformed form of $A - \lambda E$ positive realness test for $G(s)$ is reduced to testing positive realness of proper transfer function.
Method II: Case $n_2 = n_3 \neq 0$

\[ \text{rank} \left( \begin{bmatrix} A_{22} & A_{23} - \lambda E_{23} \\ 0 & A_{33} \end{bmatrix} \right) = n_2 + n_3 \forall \lambda \in \mathbb{C} \implies A_{22} \text{ and } A_{33} \text{ nonsingular.} \]

\[ \implies \text{New representation of } G_p \text{ and } M_1: \]

\[ G_p(s) = \begin{bmatrix} 0 & 0 & C_1 & C_2 & C_3 & C_4 \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & E_{23} & E_{24} \\ 0 & 0 & E_{11} & E_{12} & E_{13} & E_{14} \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} -E_{11} & -E_{12} & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & 0 & 0 \\ A_{11} & A_{12} & A_{11} & A_{12} & A_{13} & A_{14} \\ 0 & 0 & 0 & A_{22} & A_{23} & A_{24} \\ 0 & 0 & 0 & 0 & A_{33} & A_{34} \\ 0 & 0 & 0 & 0 & 0 & A_{44} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} \]

\[ M_1 = \begin{bmatrix} C_1 & C_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_{11} & E_{12} & 0 & 0 \\ 0 & -A_{22} & E_{23} & E_{24} \\ 0 & 0 & -A_{33} & -A_{34} \\ 0 & 0 & 0 & -A_{44} \end{bmatrix}^{-1} \begin{bmatrix} 0 \\ 0 \\ B_3 \\ B_4 \end{bmatrix}. \]
Method II: Case $n_2 = n_3 \neq 0$

$M_1 \geq 0$: using a sequence of RQ/QR/QR factorizations, we get

$$M_1 = \mathcal{N}^{-1} \mathcal{M}, \quad \mathcal{N}, \mathcal{M} \in \mathbb{R}^{m \times m}.$$

$$\implies M_1 \geq 0 \iff MN^T \geq 0$$

$G_p(s)$ positive real: using a sequence of QR/RQ factorizations, we get

$$G_p(s) = \begin{bmatrix} C_1 & C_2 \end{bmatrix} \left( s \begin{bmatrix} \varepsilon_{11} & 0 \\ 0 & 0 \end{bmatrix} - \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \right)^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} + \mathcal{D},$$

with $\varepsilon_{11}, A_{22}$ nonsingular.

$$\implies \text{Positive realness of } G_p(s) \text{ reduced to positive realness test for proper transfer function in standard form.}$$
Conclusions and Outlook

- \(O(n^3)\) numerical algorithm for reducing the passivity test for descriptor systems to passivity test for minimal, proper transfer functions, using orthogonal RSE only.

- Based on the special form of proper systems to be tested for positive realness, we also have derived a special recursive orthogonal reduction procedure to ARE-based “strict positive realness” test.

- Implementation and thorough numerical testing necessary.

- Usage of a priori knowledge about, e.g., index or nullspace of \(E\), can improve the performance of the algorithms.

- Method I suitable if index-1 or properness can be assumed.

- None of the approaches applicable to large, sparse descriptor systems, but sufficient for testing positive realness in model reduction methods based on modal truncation and Padé-type approximation.

- Sometimes, Writing down an LMI is not the end in computational control.