# $\mathcal{H}_2$-optimal model reduction of parametric elastic bodies

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For the efficient simulation of mechanical systems the method of elastic multibody systems is frequently used. This approach, described by Schwertassek and Wallrapp, enables the description of large nonlinear rigid body motions as well as elastic deformations which is of increasing importance in many engineering fields, e.g. robotics, automotive and power engineering. Due to the fine spatial discretization of the elastic body for the Finite Element Method, linear model order reduction techniques are often applied to the arising second order system

$$\begin{aligned} \mathbf{M}_e \cdot \ddot{\mathbf{q}}(t) + \mathbf{D}_e \cdot \dot{\mathbf{q}}(t) + \mathbf{K}_e \cdot \mathbf{q}(t) &= \mathbf{B}_e \cdot \mathbf{u}(t), \\ \mathbf{y}(t) &= \mathbf{C}_e \cdot \mathbf{q}(t). \end{aligned} \tag{1}$$

The $\mathcal{H}_2$-optimal reduction with the Iterative Rational Krylov Algorithm (IRKA), described by Gugercin, Antoulas and Beattie, enables the determination of a reduced system which provides, at least, locally $\mathcal{H}_2$-norm optimal approximations to the original system. This approach is extended by Baur et al. to parametric systems with linear parameter dependency $\mathbf{p}$ only in the input-to-state matrix $\mathbf{b}(\mathbf{p}) = \mathbf{b}_0 + p_1\mathbf{b}_1$, and in the state-output-matrix $\mathbf{c}(\mathbf{p}) = \mathbf{c}_0 + p_2\mathbf{c}_1$ by defining a composite $\mathcal{H}_2 \otimes \mathcal{L}_2$-system norm

$$||\mathbf{H}||^2_{\mathcal{H}_2\otimes\mathcal{L}_2(D)} = \frac{1}{2\pi}\int_{-\infty}^{\infty}\iint_D |\mathbf{H}(i\omega,\mathbf{p})|^2 dA(\mathbf{p})d\omega \tag{2}$$

and enables producing a locally optimal parametric model in this composite error measure.

The force position on an elastic body can vary and is described with a parameter dependent input matrix $\mathbf{b}_e(\mathbf{p})$. In this contribution, the method described by Baur et al. is extended for mechanical systems with the transfer function

$$\mathbf{H}(s,\mathbf{p}) = \mathbf{c}_e(\mathbf{p}) \cdot (s^2\mathbf{M}_e + s\mathbf{D}_e + \mathbf{K}_e)^{-1} \cdot \mathbf{b}_e(\mathbf{p}) \tag{3}$$

to minimize the $\mathcal{H}_2 \otimes \mathcal{L}_2$-norm. The direct application of IRKA for second order systems and the investigation of mechanical systems of varying size and complexity will be presented. The quality of the reduced system is determined in comparison to reduction techniques based on the Component Mode Synthesis and Gramian matrices, which are described for elastic multibody systems by Fehr.

The definition of the parameter dependent input matrix for the force varying position problem $\mathbf{b}_e(p) = \sum_i^k \omega_i(p)\mathbf{b}_{e,i}$ with weighting functions $\omega_i$ requires an additional adaption of the calculation of the alternative weighted MIMO system.

References:

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