Model order reduction and Dynamic Iteration for coupled systems

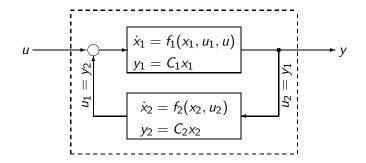
Johanna Kerler joint work with Tatjana Stykel

University of Augsburg

12.12.2013

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Model order reduction and Dynamic Iteration for coupled systems



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Problems:

whole system is very large

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- whole system is very large
- different properties of subsystems

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- whole system is very large
- different properties of subsystems

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different time scales

- whole system is very large
- different properties of subsystems

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different time scales

Approaches:

- whole system is very large
- different properties of subsystems

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different time scales

Approaches:

model order reduction

- whole system is very large
- different properties of subsystems

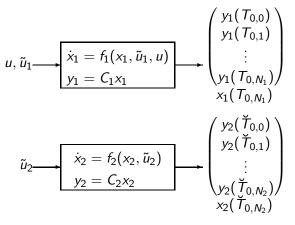
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different time scales

Approaches:

- model order reduction
- Dynamic Iteration

Dynamic Iteration - 1st macro step - initial step



initial step on $[T_0, T_1]$

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Dynamic Iteration - 1st macro step - initial step

 $u, \tilde{u}_{1} \longrightarrow \begin{bmatrix} \dot{x}_{1} = f_{1}(x_{1}, \tilde{u}_{1}, u) \\ y_{1} = C_{1}x_{1} \end{bmatrix} \longrightarrow \begin{bmatrix} y_{1}(T_{0,0}) \\ y_{1}(T_{0,1}) \\ \vdots \\ y_{1}(T_{0,N_{1}}) \end{bmatrix}$ approximate $\tilde{u}_1 \equiv y_2(T_0)$ and $x_1(T_{0,N_1})$ $\tilde{u}_2 \equiv y_1(T_0)$ $\dot{x}_2 = f_2(x_2, \tilde{u}_2)$ $y_2 = C_2 x_2$

initial step on $[T_0, T_1]$

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Dynamic Iteration - 1st macro step - initial step

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• approximate $\tilde{u}_1 \equiv y_2(T_0)$ and $\tilde{u}_2 \equiv y_1(T_0)$

 solve the systems separately

$$\tilde{u}_{1} \longrightarrow \begin{array}{c} \dot{x}_{1} = f_{1}(x_{1}, \tilde{u}_{1}, u) \\ y_{1} = C_{1}x_{1} \end{array} \longrightarrow \begin{array}{c} \begin{pmatrix} y_{1}(T_{0,0}) \\ y_{1}(T_{0,1}) \\ \vdots \\ y_{1}(T_{0,N_{1}}) \end{pmatrix} \\ x_{1}(T_{0,N_{1}}) \\ x_{1}(T_{0,N_{1}}) \end{pmatrix} \\ \tilde{u}_{2} \longrightarrow \begin{array}{c} \dot{x}_{2} = f_{2}(x_{2}, \tilde{u}_{2}) \\ y_{2} = C_{2}x_{2} \end{array} \longrightarrow \begin{pmatrix} y_{2}(\breve{T}_{0,0}) \\ y_{2}(\breve{T}_{0,1}) \\ \vdots \\ y_{2}(\breve{T}_{0,N_{2}}) \end{pmatrix} \\ x_{2}(\breve{T}_{0,N_{2}}) \end{pmatrix}$$

initial step on $[T_0, T_1]$

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Dynamic Iteration - 1st macro step - initial step

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- approximate $\tilde{u}_1 \equiv y_2(T_0)$ and $\tilde{u}_2 \equiv y_1(T_0)$
- solve the systems separately
- store y₁ and y₂ at all micro steps

$$\widetilde{u}_{1} \longrightarrow \overbrace{x_{1} = f_{1}(x_{1}, \widetilde{u}_{1}, u)}^{\dot{x}_{1} = f_{1}(x_{1}, \widetilde{u}_{1}, u)} \longrightarrow \left(\begin{array}{c}y_{1}(T_{0,0})\\y_{1}(T_{0,1})\\\vdots\\y_{1}(T_{0,N_{1}})\right)\\x_{1}(T_{0,N_{1}})\\x_{1}(T_{0,N_{1}})\\\vdots\\y_{2} = C_{2}x_{2}\end{array}\right) \longrightarrow \left(\begin{array}{c}y_{2}(\breve{T}_{0,0})\\y_{2}(\breve{T}_{0,1})\\\vdots\\y_{2}(\breve{T}_{0,N_{2}})\\x_{2}(\breve{T}_{0,N_{2}})\right)\\x_{2}(\breve{T}_{0,N_{2}})\end{array}\right)$$

initial step on $[T_0, T_1]$

Dynamic Iteration - 1st macro step - iteration step

- approximate u₁
 and u₂ by
 interpolation of y₁
 and y₂ from the
 last iteration
- solve the subsystems separately
- store y₁ and y₂ at all micro steps

$$u, \tilde{u}_{1} \longrightarrow \begin{bmatrix} \dot{x}_{1} = f_{1}(x_{1}, \tilde{u}_{1}, u) \\ y_{1} = C_{1}x_{1} \end{bmatrix} \longrightarrow \begin{bmatrix} y_{1}(T_{0,0}) \\ y_{1}(T_{0,1}) \\ \vdots \\ y_{1}(T_{0,N_{1}}) \\ x_{1}(T_{0,N_{1}}) \end{bmatrix}$$
$$\tilde{u}_{2} \longrightarrow \begin{bmatrix} \dot{x}_{2} = f_{2}(x_{2}, \tilde{u}_{2}) \\ y_{2} = C_{2}x_{2} \end{bmatrix} \longrightarrow \begin{bmatrix} y_{2}(\breve{T}_{0,0}) \\ y_{2}(\breve{T}_{0,1}) \\ \vdots \\ y_{2}(\breve{T}_{0,N_{2}}) \\ x_{2}(\breve{T}_{0,N_{2}}) \end{bmatrix}$$

iteration step on $[T_0, T_1]$

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Dynamic Iteration - i-th macro step - initial step

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- approximate u₁
 and u₂ by
 extrapolation of y₁
 and y₂
- solve the subsystems separately
- store y₁ and y₂ at all micro steps

$$\tilde{u}_{1} \xrightarrow{\qquad } \dot{x}_{1} = f_{1}(x_{1}, \tilde{u}_{1}, u) \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ y_{1}(T_{i,1}) \\ \vdots \\ y_{1} = C_{1}x_{1} \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ y_{1}(T_{i,1}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,N_{1}}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\ \vdots \\ y_{1}(T_{i,0}) \end{pmatrix} \xrightarrow{\qquad } \begin{pmatrix} y_{1}(T_{i,0}) \\$$

$$\tilde{u}_{2} \longrightarrow \begin{array}{c} \dot{x}_{2} = f_{2}(x_{2}, \tilde{u}_{2}) \\ y_{2} = C_{2}x_{2} \end{array} \longrightarrow \begin{array}{c} \begin{pmatrix} y_{2}(\breve{T}_{i,1}) \\ y_{2}(\breve{T}_{i,N_{2}}) \\ \vdots \\ y_{2}(\breve{T}_{i,N_{2}}) \end{pmatrix} \\ x_{2}(\breve{T}_{i,N_{2}}) \end{array}$$

initial step on $[T_i, T_{i+1}]$

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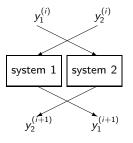
Dynamic Iteration - i-th macro step - iteration step

- approximate u₁
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$$\tilde{u}_{2} \longrightarrow \begin{bmatrix} \dot{x}_{2} = f_{2}(x_{2}, \tilde{u}_{2}) \\ y_{2} = C_{2}x_{2} \end{bmatrix} \longrightarrow \begin{bmatrix} y_{2}(\breve{T}_{i,0}) \\ y_{2}(\breve{T}_{i,1}) \\ \vdots \\ y_{2}(\breve{T}_{i,N_{2}}) \end{pmatrix}$$
$$x_{2}(\breve{T}_{i,N_{2}})$$

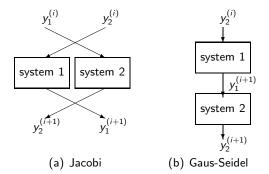
iteration step on $[T_i, T_{i+1}]$

Communication between the subsystems



(a) Jacobi

Communication between the subsystems



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Known theory:

► for ODEs:

Known theory:

- for ODEs:
 - converges for all systems if macro step size is small enough and mild assumptions for the data flow

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error bounds are known

Known theory:

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- error bounds are known
- step-size controller

Known theory:

- for ODEs:
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- error bounds are known
- step-size controller
- preconditioning to speed up the convergence

Known theory:

- for ODEs:
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- error bounds are known
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► for DAEs:

Known theory:

- for ODEs:
 - converges for all systems if macro step size is small enough and mild assumptions for the data flow
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 - step-size controller
 - preconditioning to speed up the convergence
- ► for DAEs:
 - converges not for all systems. Convergence depends on the system and on the data flow

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Known theory:

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step-size controller

Known theory:

- for ODEs:
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 - step-size controller
 - preconditioning to speed up the convergence
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- step-size controller
- regularization to enforce the convergence

DIRM

[RATHINAM/PETZOLD '2002]

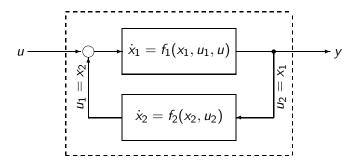
Idea: combine model reduction with Dynamic Iteration



DIRM

[RATHINAM/PETZOLD '2002]

Idea: combine model reduction with Dynamic Iteration



DIRM - 1st macro step - initial step

• approximate $\tilde{u}_1 \equiv x_2(T_0)$ and $\tilde{u}_2 \equiv x_1(T_0)$

- solve the systems separately
- store x₁ and x₂ at all micro steps

$$u, \tilde{u}_{1} \longrightarrow \dot{x}_{1} = f_{1}(x_{1}, \tilde{u}_{1}, u) \longrightarrow \begin{pmatrix} x_{1}(T_{0,0}) \\ x_{1}(T_{0,1}) \\ \vdots \\ x_{1}(T_{0,N_{1}}) \end{pmatrix}$$
$$\tilde{u}_{2} \longrightarrow \dot{x}_{2} = f_{2}(x_{2}, \tilde{u}_{2}) \longrightarrow \begin{pmatrix} x_{2}(\breve{T}_{0,0}) \\ x_{2}(\breve{T}_{0,1}) \\ \vdots \\ x_{2}(\breve{T}_{0,N_{2}}) \end{pmatrix}$$

Initial step on $[T_0, T_1]$

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DIRM - 1st macro step - iteration step

$$u \longrightarrow \dot{x}_{1} = f_{1}(x_{1}, \hat{u}_{1}, u) \longrightarrow \begin{pmatrix} x_{1}(T_{0,0}) \\ x_{1}(T_{0,1}) \\ \vdots \\ x_{1}(T_{0,N_{1}}) \end{pmatrix}$$
$$u \longrightarrow \dot{x}_{1} = \hat{f}_{1}(\hat{x}_{1}, u_{1}, u) \longrightarrow \begin{pmatrix} x_{2}(\breve{T}_{0,0}) \\ x_{2}(\breve{T}_{0,1}) \\ \vdots \\ x_{2}(\breve{T}_{0,N_{2}}) \end{pmatrix}$$

Iteration step on $[T_0, T_1]$

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DIRM - 1st macro step - iteration step

 calculate the reduced-order subsystems from snapshots x₁ and x₂ from the last iteration

$$u \xrightarrow{\dot{x}_{1} = f_{1}(x_{1}, \hat{u}_{1}, u)} \xrightarrow{\dot{x}_{2} = \hat{f}_{2}(\hat{x}_{2}, u_{2})} \xrightarrow{\begin{pmatrix} x_{1}(T_{0,0}) \\ x_{1}(T_{0,1}) \\ \vdots \\ x_{1}(T_{0,N_{1}}) \end{pmatrix}} \\ u \xrightarrow{\dot{x}_{2} = \hat{f}_{2}(\hat{x}_{2}, u_{2})} \xrightarrow{\dot{x}_{2} = f_{2}(x_{2}, \hat{u}_{2})} \xrightarrow{\begin{pmatrix} x_{2}(\breve{T}_{0,0}) \\ x_{2}(\breve{T}_{0,1}) \\ \vdots \\ x_{2}(\breve{T}_{0,N_{2}}) \end{pmatrix}}$$

Iteration step on $[T_0, T_1]$

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DIRM - 1st macro step - iteration step

- calculate the reduced-order subsystems from snapshots x₁ and x₂ from the last iteration
- solve every subsystem coupled with other reduced subsystems

$$u \longrightarrow \dot{x}_{1} = f_{1}(x_{1}, \hat{u}_{1}, u) \longrightarrow \begin{pmatrix} x_{1}(T_{0,0}) \\ x_{1}(T_{0,1}) \\ \vdots \\ x_{1}(T_{0,N_{1}}) \end{pmatrix}$$
$$u \longrightarrow \dot{\hat{x}}_{2} = \hat{f}_{2}(\hat{x}_{2}, u_{2}) \longrightarrow \begin{pmatrix} x_{2}(\breve{T}_{0,0}) \\ x_{2}(\breve{T}_{0,1}) \\ \vdots \\ x_{2}(\breve{T}_{0,N_{2}}) \end{pmatrix}$$

Iteration step on $[T_0, T_1]$

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DIRM - 1st macro step - iteration step

- calculate the reduced-order subsystems from snapshots x₁ and x₂ from the last iteration
- solve every subsystem coupled with other reduced subsystems
- store x₁ and x₂

Iteration step on $[T_0, T_1]$

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DIRM - i-th macro step - initial step

approximate u₁
 and u₂ by
 extrapolation of x₂
 and x₁

$$u, \tilde{u}_1 \longrightarrow \dot{x}_1 = f_1(x_1, \tilde{u}_1, u) \longrightarrow \begin{pmatrix} x_1(T_{i,0}) \\ x_1(T_{i,1}) \\ \vdots \\ x_1(T_{i,N_1}) \end{pmatrix}$$

- solve the systems separately
- store x₁ and x₂ at all micro steps

$$\tilde{u}_{2} \longrightarrow \dot{x}_{2} = f_{2}(x_{2}, \tilde{u}_{2}) \longrightarrow \begin{pmatrix} x_{2}(\breve{T}_{i,0}) \\ x_{2}(\breve{T}_{i,1}) \\ \vdots \\ x_{2}(\breve{T}_{i,N_{2}}) \end{pmatrix}$$

Initial step on $[T_i, T_{i+1}]$

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DIRM - i-th macro step - iteration step

- calculate the reduced-order subsystems from snapshots x₁ and x₂ from last iteration
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$$u \longrightarrow \dot{x}_{1} = f_{1}(x_{1}, \hat{u}_{1}, u) \longrightarrow \begin{pmatrix} x_{1}(T_{i,0}) \\ x_{1}(T_{i,1}) \\ \vdots \\ x_{1}(T_{i,N_{1}}) \end{pmatrix}$$
$$u \longrightarrow \dot{\hat{x}}_{1} = \hat{f}_{2}(\hat{x}_{2}, u_{2}) \longrightarrow \begin{pmatrix} x_{2}(\breve{T}_{i,0}) \\ x_{2}(\breve{T}_{i,1}) \\ \vdots \\ x_{2}(\breve{T}_{i,N_{2}}) \end{pmatrix}$$

Iteration step on $[T_i, T_{i+1}]$

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Model order reduction and Dynamic Iteration for coupled systems

LDynamic Iteration using Reduced order Models (DIRM)

iterations S1 Ś2 P3-4 Sî S2 S1 S1 Ś2 S2 P1-3 P3-3 Ś1 S2 Sî S2 Ś2 S1 Ś2 S1 Ś2 S1 P2-2 P3-2 P1-2 Ś1 Ŝ1 S2 Sî S2 S2 S1 S2 S1 S2 S1 Ś2 P2-1 P1-1 P3-1 Ś1 S2 Ś1 S2 Sî1 S2 initial step initial step initial step +≻ t

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Model order reduction and Dynamic Iteration for coupled systems
Dynamic Iteration using Reduced order Models (DIRM)

Problems:

 convergence proof only for linear systems with "weak coupling"

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Model order reduction and Dynamic Iteration for coupled systems
Dynamic Iteration using Reduced order Models (DIRM)

Problems:

 convergence proof only for linear systems with "weak coupling"

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no error estimates

Model order reduction and Dynamic Iteration for coupled systems Dynamic Iteration using Reduced order Models (DIRM)

Problems:

- convergence proof only for linear systems with "weak coupling"
- no error estimates
- no strategy to choose macro step size, number of iterations or reduced dimension

Model order reduction and Dynamic Iteration for coupled systems

L Dynamic Iteration using Reduced order Models (DIRM)

-a-posteori error estimate for ODEs

$$\dot{x} = f(x, u)$$

 $x \in \mathbb{R}^n, \ u \in \mathbb{R}^m.$ \Rightarrow $\dot{\hat{x}} = \hat{f}(\hat{x}, u)$
 $\hat{x} \in \mathbb{R}^k, \ u \in \mathbb{R}^m.$

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Model order reduction and Dynamic Iteration for coupled systems Lynamic Iteration using Reduced order Models (DIRM)

a-posteori error estimate for ODEs

$$\begin{aligned} \dot{x} &= f(x, u) \\ x \in \mathbb{R}^n, \ u \in \mathbb{R}^m. \end{aligned} \Rightarrow \quad \begin{aligned} \dot{\hat{x}} &= \hat{f}(\hat{x}, u) \\ \hat{x} \in \mathbb{R}^k, \ u \in \mathbb{R}^m. \end{aligned}$$

Modelreduction by projection. Let $V \in \mathbb{R}^{n,k}$. Then $x \approx V\hat{x}$ where \hat{x} solves

$$\dot{\hat{x}} = V^T f(V\hat{x}, u)$$

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Error in one macro step at one arbitrary iteration

an coupled system with 2 subsystems:

$$\dot{x}_1 = f_1(x_1, x_2), \qquad \dot{x}_2 = f_2(x_1, x_2).$$

Error in one macro step at one arbitrary iteration

an coupled system with 2 subsystems:

$$\dot{x}_1 = f_1(x_1, x_2), \qquad \dot{x}_2 = f_2(x_1, x_2).$$

Start with one iteration step at one macro step *i*:

$$\dot{x}_1^{D1} = f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}), \qquad \dot{\hat{x}}_1^{D2} = \hat{f}_1(-\hat{x}_1^{D2}, x_2^{D2}), \\ \dot{\hat{x}}_2^{D1} = \hat{f}_2(x_1^{D1}, -\hat{x}_2^{D1}), \qquad \dot{x}_2^{D2} = f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}).$$

Error in one macro step at one arbitrary iteration

an coupled system with 2 subsystems:

$$\dot{x}_1 = f_1(x_1, x_2), \qquad \dot{x}_2 = f_2(x_1, x_2).$$

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$$\begin{aligned} \dot{x}_1^{D1} &= f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}), & \dot{\hat{x}}_1^{D2} &= \hat{f}_1(-\hat{x}_1^{D2}, x_2^{D2}), \\ \dot{\hat{x}}_2^{D1} &= \hat{f}_2(x_1^{D1}, -\hat{x}_2^{D1}), & \dot{x}_2^{D2} &= f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}). \end{aligned}$$
Define $e := \begin{pmatrix} x_1 - x_1^{D1} \\ x_2 - x_2^{D2} \end{pmatrix}.$

Error in one macro step at one arbitrary iteration

an coupled system with 2 subsystems:

$$\dot{x}_1 = f_1(x_1, x_2), \qquad \dot{x}_2 = f_2(x_1, x_2).$$

Start with one iteration step at one macro step *i*:

$$\begin{aligned} \dot{x}_{1}^{D1} &= f_{1}(x_{1}^{D1}, V_{2} \hat{x}_{2}^{D1}), & \dot{\hat{x}}_{1}^{D2} &= \hat{f}_{1}(\hat{x}_{1}^{D2}, x_{2}^{D2}), \\ \dot{\hat{x}}_{2}^{D1} &= \hat{f}_{2}(x_{1}^{D1}, \hat{x}_{2}^{D1}), & \dot{x}_{2}^{D2} &= f_{2}(V_{1} \hat{x}_{1}^{D2}, x_{2}^{D2}). \end{aligned}$$
Define $e := \begin{pmatrix} x_{1} - x_{1}^{D1} \\ x_{2} - x_{2}^{D2} \end{pmatrix}.$
dea: solve $\frac{d \|e\|}{dt} &= \frac{\langle \dot{e}, e \rangle}{\|e\|}$ for $\|e\|$

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Model order reduction and Dynamic Iteration for coupled systems

a-posteori error estimate for ODEs

$$\langle \dot{e}, e \rangle = \left\langle \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix}, e \right\rangle$$

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Model order reduction and Dynamic Iteration for coupled systems $\hfill Dynamic Iteration using Reduced order Models (DIRM)$

-a-posteori error estimate for ODEs

$$\begin{aligned} \langle \dot{e}, e \rangle &= \left\langle \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix}, e \right\rangle \\ &= \left\langle \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, x_2^{D2}) \\ f_2(x_1^{D1}, x_2^{D2}) \end{pmatrix}, e \right\rangle \\ &+ \left\langle \begin{pmatrix} f_1(x_1^{D1}, x_2^{D1}) \\ f_2(x_1^{D1}, x_2^{D2}) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix}, e \right\rangle \end{aligned}$$

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Model order reduction and Dynamic Iteration for coupled systems Dynamic Iteration using Reduced order Models (DIRM)

-a-posteori error estimate for ODEs

$$\begin{split} \dot{e}, e \rangle &= \left\langle \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix}, e \right\rangle \\ &= \left\langle \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, x_2^{D2}) \\ f_2(x_1^{D1}, x_2^{D2}) \end{pmatrix}, e \right\rangle \\ &+ \left\langle \begin{pmatrix} f_1(x_1^{D1}, x_2^{D1}) \\ f_2(x_1^{D1}, x_2^{D2}) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix}, e \right\rangle \\ &\leq \left\langle \begin{pmatrix} f_1(x_1, x_2) \\ f_2(x_1, x_2) \end{pmatrix} - \begin{pmatrix} f_1(x_1^{D1}, x_2^{D2}) \\ f_2(x_1^{D1}, x_2^{D2}) \end{pmatrix}, e \right\rangle \\ &+ \left\| \begin{pmatrix} f_1(x_1^{D1}, x_2^{D2}) - f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(x_1^{D1}, x_2^{D2}) - f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix} \right\| \|e\| \end{split}$$

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Logarithmic constant:

$$L_G[f] := \sup_{x \neq y \in \mathbb{R}^d} rac{\langle x - y, f(x) - f(y)
angle}{\|x - y\|^2}$$

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Logarithmic constant:

$$L_G[f] := \sup_{x \neq y \in \mathbb{R}^d} \frac{\langle x - y, f(x) - f(y) \rangle}{\|x - y\|^2}$$

Approximation the log-constant by the log-constant of the Jacobian. [WIRTZ/SORENSEN/HAASDONK '2012]

$$L_{G}[J(f)] = \sup_{\mathbb{R}^{d} \setminus \{0\}} \frac{\langle x, J(f)x \rangle}{\langle x, x \rangle} = \lambda_{\max} \left(\frac{1}{2} (J(f) + J(f)^{T}) \right)$$

Logarithmic constant:

$$L_G[f] := \sup_{x \neq y \in \mathbb{R}^d} \frac{\langle x - y, f(x) - f(y) \rangle}{\|x - y\|^2}$$

Approximation the log-constant by the log-constant of the Jacobian. [WIRTZ/SORENSEN/HAASDONK '2012]

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We can sum this up with

$$\frac{d\|e\|}{dt} \leqslant \alpha \|e\| + \beta$$

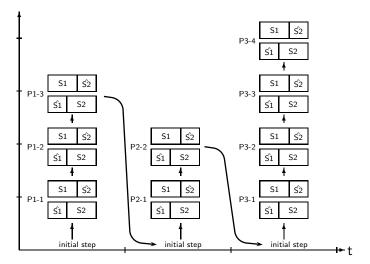
with

$$\alpha = L_G[J(f)](x_1^{D1}, x_2^{D2}),$$

$$\beta = \left\| \begin{pmatrix} f_1(x_1^{D1}, x_2^{D2}) - f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(x_1^{D1}, x_2^{D2}) - f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix} \right\|.$$

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iterations



Error for DIRM

$$\begin{aligned} \|e(T_0)\| &= 0; \\ \text{for every macro step do} \\ & \text{ in the last iteration solve} \\ & \vdots \\ & \|\dot{e}\| = \alpha \|e\| + \beta, \\ & \text{with} \\ & \alpha = L_G[J(f)](x_1^{D1}, x_2^{D2}), \\ & \beta = \left\| \begin{pmatrix} f_1(x_1^{D1}, x_2^{D2}) - f_1(x_1^{D1}, V_2 \hat{x}_2^{D1}) \\ f_2(x_1^{D1}, x_2^{D2}) - f_2(V_1 \hat{x}_1^{D2}, x_2^{D2}) \end{pmatrix} \right\|. \\ & \text{end} \end{aligned}$$

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example:

$$\dot{x} = \nu \Delta x + ax \cdot x_s$$
$$x = 0$$
$$x(0, s) = x_0$$

$\begin{array}{l} \text{in } (\Omega_1\cup\Omega_2)\times[0,\,T] \\ \text{on } \partial(\Omega_1\cup\Omega_2)\times[0,\,T] \end{array}$

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example:

$$\begin{aligned} \dot{x} &= \nu \Delta x + ax \cdot x_s & \text{in } (\Omega_1 \cup \Omega_2) \times [0, T] \\ x &= 0 & \text{on } \partial(\Omega_1 \cup \Omega_2) \times [0, T] \\ x(0, s) &= x_0 \end{aligned}$$

discretize and split in two systems:

$$\dot{x}_1 = Ax_1 + f(x_1) + b_1(x_1, x_2)$$
 $\dot{x}_2 = Ax_2 + f(x_2) + b_2(x_1, x_2)$

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dimension of subsystems: 50

example:

$$\begin{aligned} \dot{x} &= \nu \Delta x + ax \cdot x_s & \text{in } (\Omega_1 \cup \Omega_2) \times [0, T] \\ x &= 0 & \text{on } \partial(\Omega_1 \cup \Omega_2) \times [0, T] \\ x(0, s) &= x_0 \end{aligned}$$

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- dimension of subsystems: 50
- reduced dimensions: 2

example:

$$\begin{aligned} \dot{x} &= \nu \Delta x + ax \cdot x_s & \text{in } (\Omega_1 \cup \Omega_2) \times [0, T] \\ x &= 0 & \text{on } \partial(\Omega_1 \cup \Omega_2) \times [0, T] \\ x(0, s) &= x_0 \end{aligned}$$

discretize and split in two systems:

$$\dot{x}_1 = Ax_1 + f(x_1) + b_1(x_1, x_2)$$
 $\dot{x}_2 = Ax_2 + f(x_2) + b_2(x_1, x_2)$

- dimension of subsystems: 50
- reduced dimensions: 2
- number of iterations: 3

example:

$$\begin{aligned} \dot{x} &= \nu \Delta x + ax \cdot x_s & \text{in } (\Omega_1 \cup \Omega_2) \times [0, T] \\ x &= 0 & \text{on } \partial(\Omega_1 \cup \Omega_2) \times [0, T] \\ x(0, s) &= x_0 \end{aligned}$$

discretize and split in two systems:

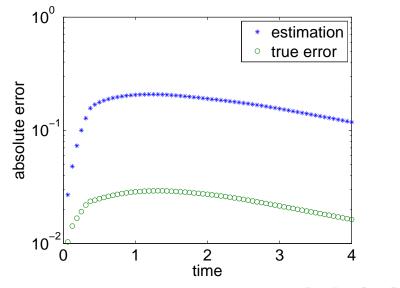
$$\dot{x}_1 = Ax_1 + f(x_1) + b_1(x_1, x_2)$$
 $\dot{x}_2 = Ax_2 + f(x_2) + b_2(x_1, x_2)$

- dimension of subsystems: 50
- reduced dimensions: 2
- number of iterations: 3
- ► $-0.3 \leq L_G(J(f))) \leq 0.01$

Model order reduction and Dynamic Iteration for coupled systems

L Dynamic Iteration using Reduced order Models (DIRM)

-a-posteori error estimate for ODEs



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Conclusion

Presented:

Dynamic Iteration

Conclusion

Presented:

Dynamic Iteration

DIRM

Conclusion

Presented:

- Dynamic Iteration
- DIRM
- a posteori error estimate

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Conclusion

Presented:

- Dynamic Iteration
- DIRM
- a posteori error estimate

Future work:

cheap computation of the error estimator (employing the structure of f? DEIM?)

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Conclusion

Presented:

- Dynamic Iteration
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Future work:

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conditions for convergence of DIRM

Conclusion

Presented:

- Dynamic Iteration
- DIRM
- a posteori error estimate

Future work:

- cheap computation of the error estimator (employing the structure of f? DEIM?)
- conditions for convergence of DIRM
- strategy to choose macro step size, reduced dimensions and number of DIRM iterations

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