Poles/residues optimization for frequency-limited $\mathcal{H}_2$ model approximation

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ModRed 2013

Introduction

Context

Various problematics arise in aeronautics

- fault detection and reconfiguration
- actuators allocation
- structured $\mathcal{H}_2$ and $\mathcal{H}_\infty$ control
- ...

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⚠Problem : the tools available for addressing these problematics are often limited by the size of the models

$\Downarrow$

Considered solution : Model approximation

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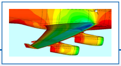
High fidelity aircraft model construction



Structure modeling

Reference model (material, masses, etc.)

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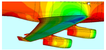

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Parameters determination (in wind tunnel) **for some flight point**

Aerodynamic behaviour modeling

Load modeling

Simulation of the load propagation (gusts, maneuvers, etc.)

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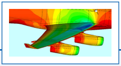

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Flight test

Adjustement of the models through identification

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Global process for aircraft control

$500 \leq n \leq 2500$

Set of **large-scale** LTI models (for each flight point)

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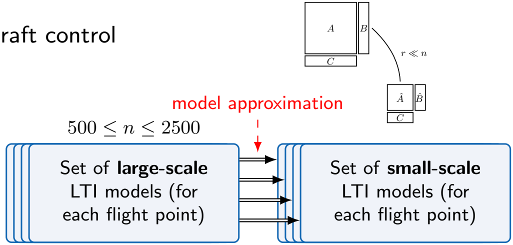
Global process for aircraft control



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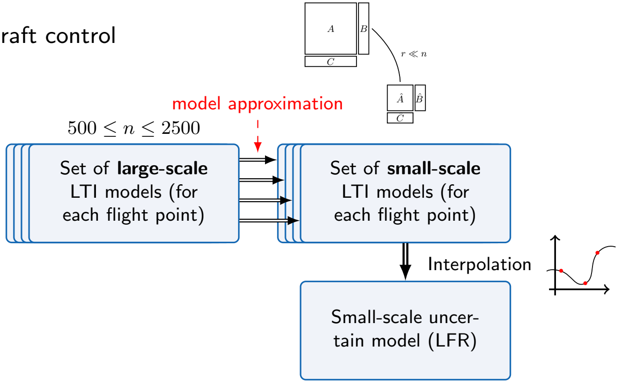

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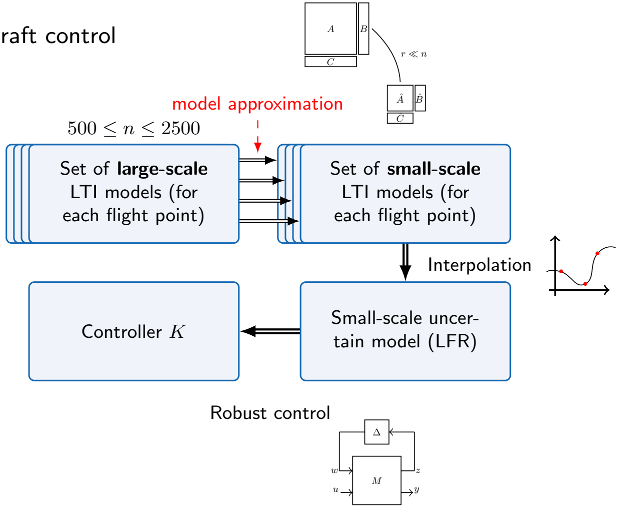

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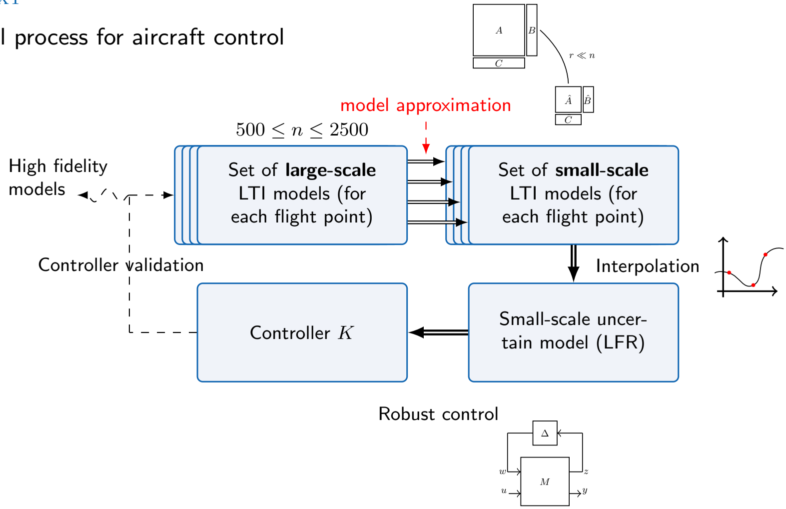

Introduction

Problem statement

$\mathcal{H}_{2,\Omega}$ approximation problem

Given a continuous LTI dynamical system H of order n, the problem consists in finding a stable r-th order ($r \ll n$) model $\hat{H}$ that minimizes the frequency-limited $\mathcal{H}_2$-norm of the approximation error $\mathcal{J}_{\mathcal{H}_{2,\Omega}}$, *i.e.*

$$\hat{H} = \arg \min_{G_r \text{ stable}} \mathcal{J}_{\mathcal{H}_{2,\Omega}}(G_r) = \arg \min_{G_r \text{ stable}} \|H - G_r\|^2_{\mathcal{H}_{2,\Omega}} \quad (1)$$

Why considering the approximation over a bounded frequency interval ?

- sensors/actuators limited bandwith $\rightarrow$ some frequencies might not be useful for control purpose.
- in aeronautics, a common specification for controller is that they act on a specific frequency interval only so that they do not impact other control laws.

Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$-norm

$\mathcal{H}_{2,\Omega}$-norm

Given a frequency interval $\Omega = [0, \omega]$, the $\mathcal{H}_{2,\Omega}$-norm of a model H, denoted $\|H\|_{\mathcal{H}_{2,\Omega}}$, is defined as the restriction of the $\mathcal{H}_2$-norm over $[-\omega, \omega]$, *i.e.*

$$\|H\|^2_{\mathcal{H}_{2,\Omega}} := \frac{1}{2\pi} \int_{-\omega}^{\omega} \mathbf{tr}\left(H(j\nu)H(-j\nu)^T\right) d\nu \quad (2)$$

- Suggested for frequency analysis of unstable systems [1]
- Strongly connected to the frequency-limited gramians [2]
- Robustness analysis [3]

1. M.R. Anderson, A. Emami-Naeni and J.H. Vincent, *"Measures of merit for multivariable flight control"*, Technical report, 1991.
2. W. Gawronski and J. Juang, *"Model reduction in limited time and frequency intervals"*, International Journal of Systems Science, 1990.
3. A. Masi, R. Wallin, A. Garulli and A. Hansson, *"Robust finite-frequency $\mathcal{H}_2$ analysis"*, CDC, 2010.

Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$-norm

$\|H\|_{\mathcal{H}_{2,\Omega}}$ can be computed :

- with the frequency-limited gramians $\mathcal{P}_\omega$ and $\mathcal{Q}_\omega$ as

$$\|H\|^2_{\mathcal{H}_{2,\Omega}} = \mathbf{tr}\left(C\mathcal{P}_\omega C^T\right) = \mathbf{tr}\left(B^T\mathcal{Q}_\omega B\right) \tag{3}$$

$\rightarrow$ This formulation has been used (in parallel of this work) to perform optimal $\mathcal{H}_{2,\Omega}$ model approximation [4].

4. D. Petersson, *"A Nonlinear Optimization Approach to H2-Optimal Modeling and Control"*, PhD thesis, Linköping University, 2013.

5. P. Vuillemin, C. Poussot-Vassal and D. Alazard, *"A Spectral Expression for the Frequency-Limited $\mathcal{H}_2$-norm"*, Available as http ://arxiv.org/abs/1211.1858, 2012.

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  $\rightarrow$ This formulation has been used (in parallel of this work) to perform optimal $\mathcal{H}_{2,\Omega}$ model approximation [4].

- if H has semi-simple poles only, then [5]

$$\|H\|^2_{\mathcal{H}_{2,\Omega}} = \sum_{i=1}^{n} -\mathbf{tr}\left(\phi_i H(-\lambda_i)^T\right) \frac{2}{\pi}\mathbf{atan}\left(\frac{\omega}{\lambda_i}\right), \tag{4}$$

  where λ_i, ϕ_i $(i = 1, \ldots, n)$ are the poles and associated residues of $H(s)$ and $\mathbf{atan}(z)$ is the principal value of the complex arctangent of $z \neq \pm j$.

4. D. Petersson, *"A Nonlinear Optimization Approach to H2-Optimal Modeling and Control"*, PhD thesis, Linköping University, 2013.

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Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$-norm

Some remarks :

- Given $\Omega = [0, \omega]$, if H is stable, then

$$\lim_{\omega \to \infty} \|H\|_{\mathcal{H}_{2,\Omega}} = \|H\|_{\mathcal{H}_2}. \tag{5}$$

- If $\Omega = \Omega_1 \cap \Omega_2$ where $\Omega_1 = [0, \omega_1]$ and $\Omega_2 = [0, \omega_2]$ with $\omega_1 < \omega_2$ then

$$\|H\|^2_{\mathcal{H}_{2,\Omega}} = \|H\|^2_{\mathcal{H}_{2,\Omega_2}} - \|H\|^2_{\mathcal{H}_{2,\Omega_1}} \tag{6}$$

- Similar expressions exist for models with a direct feedthrough D.

Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$-norm

Two upper bounds on the $\mathcal{H}_\infty$-norm

$$\|H\|_{\mathcal{H}_\infty} \leq \max_{\omega\in\mathbb{R}} \left(\pi \frac{\|H\|^2_{\mathcal{H}_{2,[0,\omega]}}}{d\omega}\right)^{\frac{1}{2}} = \max_{\omega\in\mathbb{R}} \|H(j\omega)\|_F \tag{7}$$

Grounded on the poles/residues formulation of the $\mathcal{H}_{2,\Omega}$-norm [6],

$$\boxed{\|H\|_{\mathcal{H}_\infty} \leq \underbrace{\left(\max_{\omega\in\mathbb{R}} \sum_{i=1}^{n} g_i(\omega)\right)^{\frac{1}{2}}}_{\Gamma(H)} \leq \underbrace{\left(\sum_{i=1}^{n} \max_{\omega\in\mathbb{R}} g_i(\omega)\right)^{\frac{1}{2}}}_{\bar{\Gamma}(H)}} \tag{8}$$

where $g_i(\omega)$ $(i = 1, \ldots, n)$ are rational functions of ω.

- computing $\Gamma(H)$ requires to find the maximum of a sum of rational functions
- $\bar{\Gamma}(H)$ can be found analytically

6. P. Vuillemin, C. Poussot-Vassal and D. Alazard, *"Two upper bounds on the $\mathcal{H}_\infty$-norm of LTI dynamical systems"*, Submitted, 2013.

Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$ error and first-order optimality conditions [8]

$\mathcal{H}_{2,\Omega}$ approximation error
Assuming that H and $\hat{H}$ are stables and have semi-simple poles only, *i.e.*

$$H(s) = \sum_{i=1}^{n} \frac{\phi_i}{s-\lambda_i} \quad \text{and} \quad \hat{H}(s) = \sum_{k=1}^{r} \frac{\hat{\phi}_k}{s-\hat{\lambda}_k}. \tag{9}$$

Then the approximation error $\mathcal{J}_{\mathcal{H}_{2,\Omega}}$ is given by

$$\boxed{\mathcal{J}_{\mathcal{H}_{2,\Omega}} = \|H\|^2_{\mathcal{H}_{2,\Omega}} + \|\hat{H}\|^2_{\mathcal{H}_{2,\Omega}} - \frac{2}{\pi}\sum_{i=1}^{n}\sum_{k=1}^{r} \frac{\mathbf{tr}\left(\phi_i\hat{\phi}_k^T\right)}{\lambda_i+\hat{\lambda}_k}\left(\mathbf{atan}\left(\frac{\omega}{\lambda_i}\right)+\mathbf{atan}\left(\frac{\omega}{\hat{\lambda}_k}\right)\right)} \tag{10}$$

(Very similar to the $\mathcal{H}_2$ case [7])

7. S. Gugercin, A.C. Antoulas and C. Beattie, *"$\mathcal{H}_2$ Model Reduction for Large-Scale Linear Dynamical Systems"*, SIAM Journal on Matrix Analysis and Applications, 2008.
8. P. Vuillemin, C. Poussot-Vassal and D. Alazard, *"Poles Residues Descent Algorithm for Optimal Frequency-Limited $\mathcal{H}_2$ Model Approximation"*, Submitted, 2013.

Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$ error and first-order optimality conditions

Some remarks :

- to handle MIMO cases, the reduced-order model residues $\hat{\phi}_k$ $(k = 1, \ldots, r)$ must be written as an outer product, *i.e.*

$$\hat{\phi}_k = \hat{c}_k^T \hat{b}_k, \; k = 1, \ldots, r \tag{11}$$

  where $\hat{c}_k \in \mathbb{C}^{1 \times n_y}$ and $\hat{b}_k \in \mathbb{C}^{1 \times n_u}$.

- $\|\hat{H}\|^2_{\mathcal{H}_{2,\Omega}}$ is infinite when some poles of $\hat{H}$ cross the imaginary axis $\rightarrow$ the imaginary axis acts as a *natural* barrier

Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$ error and first-order optimality conditions

Grounded on the previous formulation, the optimal $\mathcal{H}_{2,\Omega}$ approximation problem can be formulated similarly to the $\mathcal{H}_2$ case [9].

$\mathcal{H}_{2,\Omega}$ approximation problem

Given a n-th order stable MIMO LTI dynamical system H with only semi-simple poles, the optimal $\mathcal{H}_{2,\Omega}$ approximation problem consists in finding the reduced-order poles and associated residues $\hat{\lambda}_k$, $\hat{c}_k$ and $\hat{b}_k$ $(k = 1, \ldots, r)$ that minimizes $\mathcal{J}_{\mathcal{H}_{2,\Omega}}$

9. C. Beattie and S. Gugercin, *"A Trust Region Method for Optimal $\mathcal{H}_2$ Model Reduction"*, CDC, 2009.

10. L. Sorber, M. Van Barel and L. De Lathauwer, *"Unconstrained optimization of real functions in complex variables"*, SIAM J. Optim, 2012.

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- $\mathcal{J}_{\mathcal{H}_{2,\Omega}}$ is not convex
- $\mathcal{J}_{\mathcal{H}_{2,\Omega}}$ is a real function of complex variables and their conjugate $\rightarrow$ the $\mathbb{CR}$ (or Wirtinger) calculus is used to derive the first-order optimality conditions [10]
- there are $r\,(1 + n_y + n_u)$ variables $\rightarrow$ the problem is overparametrized

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9. C. Beattie and S. Gugercin, *"A Trust Region Method for Optimal $\mathcal{H}_2$ Model Reduction"*, CDC, 2009.

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Frequency-limited model approximation

$\mathcal{H}_{2,\Omega}$ error and first-order optimality conditions

The conjugate cogradients $\frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial \hat{\lambda}_m^*}$, $\frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial \hat{c}_m^*}$, $\frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial \hat{b}_m^*}$, $m = 1, \dots, r$ (and $\frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial \hat{D}}$) have been derived [11] but they have not (yet) clearly been formulated as interpolation conditions.

11. P. Vuillemin, C. Poussot-Vassal and D. Alazard, *"Poles Residues Descent Algorithm for Optimal Frequency-Limited $\mathcal{H}_2$ Model Approximation"*, Submitted, 2013.

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For instance, for $m = 1, \dots, r$

$$\begin{aligned} \frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial \hat{b}_m} &= \sum_{i=1}^{r} \frac{\hat{b}_i^T \hat{c}_i \hat{c}_m^T}{\hat{\lambda}_i + \hat{\lambda}_m} \left(\textbf{atan}\left(\frac{\omega}{\hat{\lambda}_i}\right) + \textbf{atan}\left(\frac{\omega}{\hat{\lambda}_m}\right) \right) \\ &\quad - \sum_{i=1}^{n} \frac{b_i^T c_i \hat{c}_m^H}{\lambda_i + \hat{\lambda}_m} \left(\textbf{atan}\left(\frac{\omega}{\lambda_i}\right) + \textbf{atan}\left(\frac{\omega}{\hat{\lambda}_m}\right) \right) \end{aligned} \tag{12}$$

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For instance, for $m = 1, \ldots, r$

$$\frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial \hat{b}_m} = \hat{c}_m \left[\hat{H}_\omega(-\hat{\lambda}_m) - \hat{H}(-\hat{\lambda}_m)\mathbf{atan}\left(\frac{\omega}{\hat{\lambda}_m}\right)\right] - \hat{c}_m \left[H_\omega(-\hat{\lambda}_m) - H(-\hat{\lambda}_m)\mathbf{atan}\left(\frac{\omega}{\hat{\lambda}_m}\right)\right] \tag{12}$$

where

$$H_\omega(s) = \sum_{i=1}^{n} \frac{\phi_i}{s - \lambda_i}\mathbf{atan}\left(\frac{\omega}{\lambda_i}\right) \quad \text{and} \quad \hat{H}_\omega(s) = \sum_{k=1}^{r} \frac{\hat{\phi}_k}{s - \hat{\lambda}_k}\mathbf{atan}\left(\frac{\omega}{\hat{\lambda}_k}\right) \tag{13}$$

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Frequency-limited model approximation

Optimization scheme

Require: $A \in \mathbb{R}^{n \times n}$, $B \in \mathbb{R}^{n \times n_u}$, $C \in \mathbb{R}^{n_y \times n}$, $\Omega = [0, \omega]$ with $\omega > 0$ and $r \in \mathbb{N}^*$.

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1: Compute the eigenvalues and associated eigenvectors of A to determine λ_i, c_i and b_i $i = 1, \dots, n$

2: Choose an initial point z_0 composed of $\hat{\lambda}_i^{(0)}$, $\hat{c}_i^{(0)}$, $\hat{b}_i^{(0)}$, $i = 1, \dots, r$ corresponding to a stable model.

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2: Choose an initial point z_0 composed of $\hat{\lambda}_i^{(0)}$, $\hat{c}_i^{(0)}$, $\hat{b}_i^{(0)}$, $i = 1,\dots,r$ corresponding to a stable model.
3: k $\leftarrow$ 0.
4: **while not converged do**
5: Compute the error $\mathcal{J}_{\mathcal{H}_{2,\Omega}}(z_k)$ and the associated gradient $\frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial z^*}|_{z=z_k}$.
6: Choose the descent direction $p_k = -2\frac{\partial \mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\partial z^*}|_{z=z_k}$ (BFGS in practice).
7: Choose the step length α_k such that $\mathcal{J}_{\mathcal{H}_{2,\Omega}}(z_k + \alpha_k p_k)$ satisfies the strong Wolfe conditions and such that the poles do not cross the imaginary axis.
8: Set $z_{k+1} = z_k + \alpha_k p_k$.
9: k $\leftarrow$ k+1.
10: **end while**

Frequency-limited model approximation

Optimization scheme

Require: $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n_u}$, $C \in \mathbb{R}^{n_y\times n}$, $\Omega = [0, \omega]$ with $\omega > 0$ and $r \in \mathbb{N}^*$.

Compute the eigenvalues and associated eigenvectors of A to determine λ_i, c_i and b_i $i = 1, \ldots, n$

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    Set $z_{k+1} = z_k + \alpha_k p_k$.

    k $\leftarrow$ k+1.

end while

Use $\hat{\lambda}_i^{(k)}$, $\hat{c}_i^{(k)}$, $\hat{b}_i^{(k)}$, $i = 1, \ldots, r$ to construct $\hat{A}$, $\hat{B}$ et $\hat{C}$.

[Optional] Compute $\Gamma(H - \hat{H})$ and $\bar{\Gamma}(H - \hat{H})$

Applications

Simple benchmark

LAH model, $r = 20$, $\Omega = [0, \infty]$, $100\frac{\mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\|H\|^2_{\mathcal{H}_{2,\Omega}}}$ goes from 17% to 7.8%.

Applications

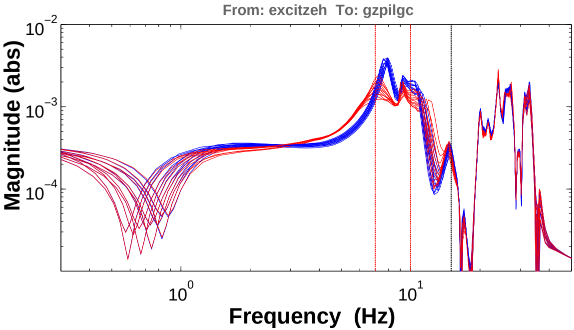
Simple benchmark

LAH model, $r = 8$, $\Omega = [0, 10]$, $100\frac{\mathcal{J}_{\mathcal{H}_{2,\Omega}}}{\|H\|^2_{\mathcal{H}_{2,\Omega}}}$ goes from 5.5% to 0.47%.

Applications

Industrial use-case

Joint work between Onera and Dassault Aviation [12]



12. C. Poussot-Vassal, C. Roos, P. Vuillemin, O. Cantinaud and J.P. Lacoste, *"Control-oriented aeroelastic BizJet low-order LFT modeling"*, To appear in *Control-oriented modeling and identification : theory and practice*, M. Lovera (Ed).

Conclusions & perspectives

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Proposed approach :

- $\oplus$ Extension of a previous approach to the frequency-limited $\mathcal{H}_2$ case.
- $\oplus$ Ensure the decrease of the $\mathcal{H}_{2,\Omega}$-error and the stability of the reduced-order model.

Conclusions & perspectives

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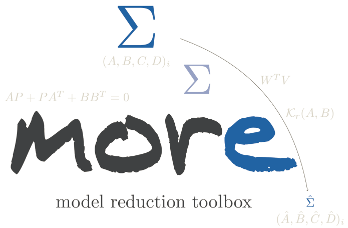
Proposed approach :

- ⊕ Extension of a previous approach to the frequency-limited $\mathcal{H}_2$ case.
- ⊕ Ensure the decrease of the $\mathcal{H}_{2,\Omega}$-error and the stability of the reduced-order model.
- ⊖ Requires the eigenvalues/eigenvectors of the initial model $\rightarrow$ dedicated to medium-scale models.
- ⊖ Local optimization algorithm $\rightarrow$ the result strongly depends on the initialization.

Conclusions & perspectives

The approach is available in the MORE Toolbox (matlab toolbox) :

http://w3.onera.fr/more/



Conclusions & perspectives

Perspectives

Concerning the algorithm :

- Implementation of a trust-region approach for (a real) constraints management
- New initialization strategy
- Sparse case ?

Then,

- Using uncertainties to model the approximation error (LFR) for control purposes.
- Combining the approximation and interpolation steps for multiples LTI models approximation (already addressed by D.Petersson with the gramian-based formulation).

Thank you for your attention !
Vielen Dank für Ihre Aufmerksamkeit !