GNAT for MOR of electrical networks with semiconductors

Michael Hinze, Ulrich Matthes

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Outline

Coupled circuit and semiconductor models

MOR of semiconductors modeled by PDEs

GNAT compared to POD DEIM

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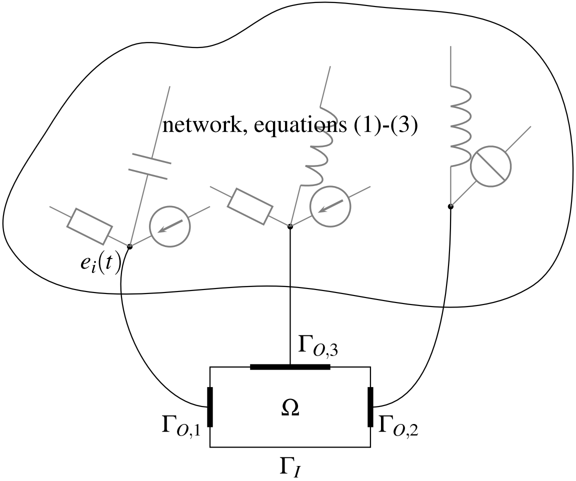
Coupled circuit and semiconductor models; sketch

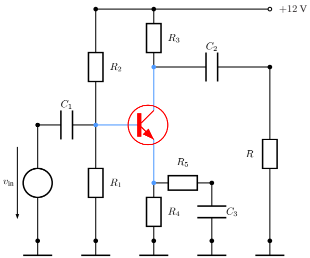


Coupled circuit and semiconductor models [M. Günther '01, C. Tischendorf '03]

Kirchhoff's' laws (no semiconductors) read

$$Aj = 0, \quad v = A^\top e$$

A: (reduced) incidence matrix.



Voltage-current relations of components:

$$j_C = \frac{\mathrm{d}q_C}{\mathrm{d}t}(v_C, t), \; j_R = g(v_R, t), \; v_L = \frac{\mathrm{d}\phi_L}{\mathrm{d}t}(j_L, t)$$

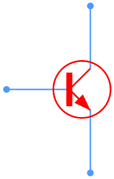
Modified Nodal Analysis: join all equations to DAE system

$$\begin{aligned} A_C \frac{\mathrm{d}q_C}{\mathrm{d}t}\left(A_C^\top e(t), t\right) + A_R g\left(A_R^\top e(t), t\right) + A_L j_L(t) + A_V j_V(t) &= -A_I i_s(t), \\ \frac{\mathrm{d}\phi_L}{\mathrm{d}t}\left(j_L(t), t\right) - A_L^\top e(t) &= 0, \\ A_V^\top e(t) &= v_s(t). \end{aligned}$$

Semiconductor modeled as PDE system

PDE-model (drift-diffusion equations) for semiconductors

$$\begin{aligned}
\operatorname{div}\,(\varepsilon\nabla\psi) &= q(n-p-C),\\
-q\partial_t n + \operatorname{div} J_n &= \quad qR(n,p),\\
q\partial_t p + \operatorname{div} J_p &= -qR(n,p),\\
J_n &= \mu_n q(\quad U_T\nabla n - n\nabla\psi),\\
J_p &= \mu_p q(-U_T\nabla p - p\nabla\psi),
\end{aligned}$$



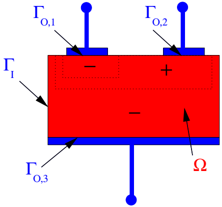
on $\Omega \times [0,T]$ with $\Omega \subset \mathbb{R}^d$ $(d = 1,2,3)$.
Dirichlet boundary constraints at $\Gamma_{O,k}$:

$$\psi(t,x), \quad n(t,x) = \tilde{n}(x), \quad p(t,x) = \tilde{p}(x)$$

and Neumann boundary constraints at Γ_I:

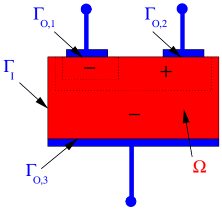
$$\nabla\psi(t,x)\cdot\nu(x) = J_n\cdot\nu(x) = J_p(t,x)\cdot\nu(x) = 0$$

or mixed boundary conditions at MI contacts (MOSFETs).



Coupling of semiconductors to the network [M. Günther '01, C. Tischendorf '03]

Coupling conditions:



$$j_{S,k}(t) = \int_{\Gamma_{O,k}} (J_n + J_p - \varepsilon \partial_t \nabla \psi) \cdot \nu \, d\sigma,$$

$$\psi(t,x) = \psi_{bi}(x) + (A_S^\top e(t))_k \quad \text{for } (t,x) \in [0,T] \times \Gamma_{O,k},$$

and add current j_S to Kirchhoff's current law:

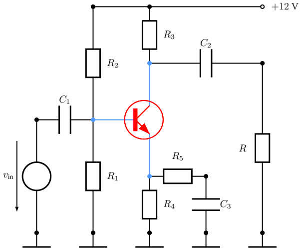
$$A_C \frac{\mathrm{d}q_C}{\mathrm{d}t}\left(A_C^\top e, t\right) + A_R g\left(A_R^\top e, t\right) + A_L j_L + A_V j_V + A_S j_S = -A_I i_s,$$

$$\frac{\mathrm{d}\phi_L}{\mathrm{d}t}(j_L, t) - A_L^\top e = 0,$$

$$A_V^\top e = v_s.$$

Add DD-equations + coupling conditions for each semiconductor.

Full model



$$
\begin{aligned}
A_C \frac{\mathrm{d}q_C}{\mathrm{d}t}\left(A_C^\top e(t), t\right) + A_R g\left(A_R^\top e(t), t\right)& \\
+A_L j_L(t) + A_V j_V(t) + A_S j_S(t) &= -A_I i_s(t), \\
\frac{\mathrm{d}\phi_L}{\mathrm{d}t}(j_L(t), t) - A_L^\top e(t) &= 0, \\
A_V^\top e(t) &= v_s(t), \\
j_S(t) - C_1 J_n(t) - C_2 J_p(t) - C_3 \dot{g}_\psi(t) &= 0,
\end{aligned}
$$

$$
\begin{pmatrix} 0 \\ -M_L \dot{n}(t) \\ M_L \dot{p}(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{FEM} \begin{pmatrix} \psi(t) \\ n(t) \\ p(t) \\ g_\psi(t) \\ J_n(t) \\ J_p(t) \end{pmatrix} + \mathcal{F}(n^h, p^h, g_\psi^h) - b(A_S^\top e(t)) = 0.
$$

first space discretization (mixed FEM), then time discretization (DASPK)

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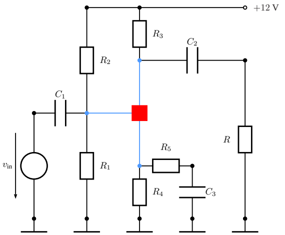
Reduced model recoupled to the network



$$
\begin{aligned}
A_C \frac{\mathrm{d}q_C}{\mathrm{d}t}\left(A_C^\top e(t), t\right) + A_R g\left(A_R^\top e(t), t\right) & \\
+ A_L j_L(t) + A_V j_V(t) + A_S j_S(t) &= -A_I i_s(t), \\
\frac{\mathrm{d}\phi_L}{\mathrm{d}t}(j_L(t), t) - A_L^\top e(t) &= 0, \\
A_V^\top e(t) &= v_s(t), \\
j_S(t) - C_1 U_{J_n}\gamma_{J_n}(t) - C_2 U_{J_p}\gamma_{J_p}(t) - C_3 U_{g_\psi}\dot{\gamma}_{g_\psi}(t) &= 0,
\end{aligned}
$$

$$
\begin{pmatrix} 0 \\ -\dot{\gamma}_n(t) \\ \dot{\gamma}_p(t) \\ 0 \\ 0 \\ 0 \end{pmatrix} + A_{POD} \begin{pmatrix} \gamma_\psi(t) \\ \gamma_n(t) \\ \gamma_p(t) \\ \gamma_{g_\psi}(t) \\ \gamma_{J_n}(t) \\ \gamma_{J_p}(t) \end{pmatrix} + U^\top \mathcal{F}(n^{POD}, p^{POD}, g_\psi^{POD}) - U^\top b(A_S^\top e(t)) = 0.
$$

FEM using reduced nonlocal basis obtained by Snapshot-POD

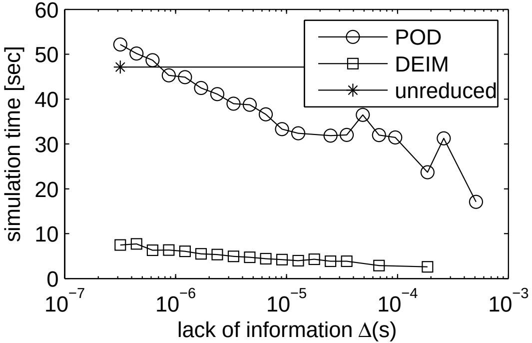
Discrete Empirical Interpolation Method (DEIM)

Reduced nonlinearity, classical treatment

$$U^\top F(U_n\gamma_n, U_p\gamma_p, U_{g_\psi}\gamma_{g_\psi})$$

with DEIM approximated as:

$$\underbrace{(U^\top W(P^\top W)^{-1})}_{n_{POD}\times n_{DEIM},\ \text{block-dense}} \quad \underbrace{P^\top}_{n_{DEIM}} F(\underbrace{U_n\gamma_n}_{n_{FEM}}, U_p\gamma_p, U_{g_\psi}\gamma_{g_\psi})$$



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Gauss-Newton with Approximated Tensors (GNAT)

K. Carlberg, et. al.: *The GNAT nonlinear model reduction method and its application to fluid dynamics problems*, in AIAA (2012)

- GNAT is a nonlinear model order reduction method
- Uses least-squares Petrov-Galerkin projection
- approximating the residual and Jacobian using the ”Gappy POD” method to reduce complexity
- time discrete approach

GNAT: Model Hierarchy

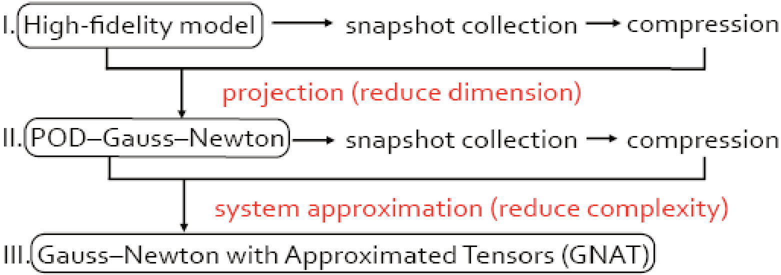


Figure 1. Model hierarchy with approximations shown in red.

Source: K. Carlberg, et. al., *The GNAT nonlinear model reduction method and its application to fluid dynamics problems*, in AIAA (2012)

GNAT: GN Tier II-Model

ODE as result of a semidiscretization of a time-dependent (parabolic) PDE

$$\dot{y}(t) = F(y(t), t; \boldsymbol{\mu}) \qquad y(0) = y_0(\boldsymbol{\mu}).$$

Implicit time integration yields a sequence of nonlinear problems from Tier I of the form

$$R^n(y^{n+1}; \boldsymbol{\mu}) = 0 \qquad \rightarrow \qquad R(y) = 0.$$

Solve, take snapshots, compute POD basis Φ_y, approximate y in the form

$$y = y^{(0)} + \Phi_y y_r$$

Solve least-squares problem

$$\min_{y \in y^{(0)} + Y} \|R(y)\|_2$$

with Gauss-Newton

$$p^{(k)} = \arg\min_{a \in R^{n_y}} \|J^{(k)} \Phi_y a + R^{(k)}\|_2$$

$$y_r^{(k+1)} = y_r^{(k)} + \alpha^{(k)} p^{(k)}$$

Classical approach: Petrov-Galerkin projection

$$\Phi_y^T R(y^{(0)} + \Phi_y y_r) = 0$$

GNAT: Tier III-Model

Gappy POD, approximating $R^{(k)}$ and $J^{(k)}\Phi_y$ by computing only a selection of their rows. $\hat{\cdot}$ denotes the restriction operator to the sample indices, Φ_R, Φ_J POD-bases

$$R^{(k)} \approx \Phi_R R_r^{(k)} \qquad J^{(k)}\Phi_y \approx \Phi_J J_r^{(k)}$$

$$R_r^{(k)} = \underset{z \in \mathbb{R}^{n_R}}{\arg\min} \, \|\hat{R}^{(k)} - \hat{\Phi}_R z\|_2$$

$$J_r^{(k)} = \underset{z \in \mathbb{R}^{n_J \times n_y}}{\arg\min} \, \|\widehat{J^{(k)}\Phi_y} - \hat{\Phi}_J z\|_2$$

Gauss-Newton with approximated tensors

$$p^{(k)} = \underset{a \in R^{n_y}}{\arg\min} \, \|\hat{\Phi}_J^{+} \widehat{J^{(k)}\Phi_y} a + \Phi_J^T \Phi_R \hat{\Phi}_R^{+} \hat{R}^{(k)}\|_2$$

$$y_r^{(k+1)} = y_r^{(k)} + \alpha^{(k)} p^{(k)}$$

$\cdot^{+}$ denotes the pseudo (left)-inverse.

A Simple Example to Test GNAT

Semilinear heat conduction equation

$$\dot{y} - \Delta y + y^3 = 0 \quad \text{in } \Omega = [0,1], \quad y_x(t,0) = y_x(t,1) = 0, \quad y(0,x) = y_0(x),$$

where y_0 is the linear B-spline, with $y_0(0.5) = 1$ and $y_0(x_i) = 0$ on the other grid points as initial condition.
As a test example for GNAT with different norms: l^2-norm and H^{-1}-norm
Time discretization: trapezoidal rule 1024 time steps
The absolute error of the 0 function would be 6.5693.

m	POD	GN-T2 l^2	GN-T2 H^{-1}	GNAT-T3 l^2	GNAT-T3 H^{-1}
1	3.1122	3.2468	3.1023	3.2468	3.1247
2	2.5975	2.7838	2.5671	2.7842	2.5675
3	2.2933	2.8680	2.9654	3.0028	2.1774
4	1.6844	2.1068	1.3497	2.1243	1.3429
5	1.1871	1.5485	1.1526	1.6895	0.9978
6	0.7795	0.9980	0.6094	1.0006	0.6003
7	0.5105	0.6681	0.4001	0.6719	0.3974
8	0.3278	0.4288	0.2520	0.4253	0.2518
9	0.2096	0.2771	0.1615	0.3060	0.1636
10	0.1326	0.1764	0.1026	0.1767	0.1028

A Simple Circuit

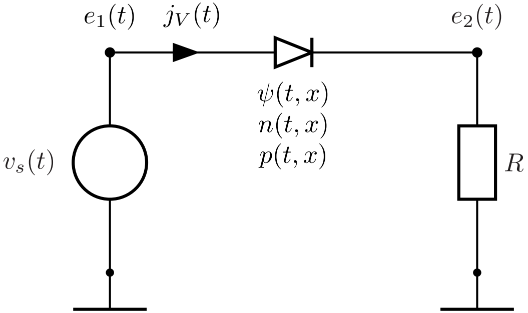
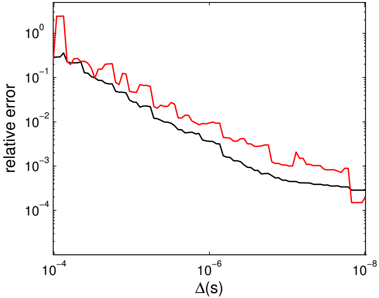


Figure: Basic circuit with one diode.

A Simple Circuit: GNAT Tier III vs. POD-DEIM



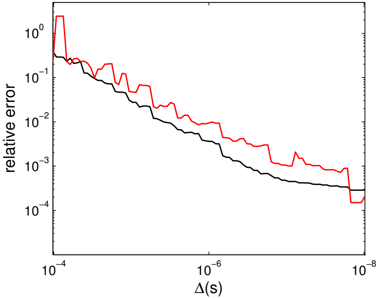


Figure: POD (red), mor reduction with GNAT Tier III (black) with initialization from last step (left), the same, but GN starts with POD solution (right), l^2-Norm of the weighted equations

Discussion: Comparing GNAT with POD-DEIM

Time discretization:

- POD-DEIM: reduced system not discretized in time; $\rightarrow$ can choose appropriate solver for the ODE/DAE system, with automatic order control and time stepping $+$
- GNAT:
  - first discretize in time, then reduce; $\rightarrow$ the operator R depends on the time discretization scheme
  - complex implementation if adaptive time integration with order control is used $-$
  - if the full system is solved with a higher order method in time, the use of implicit Euler for the GNAT-reduced system only is slow, because the low order of Euler requires to incorporate many time steps to achieve a comparable accuracy; $\rightarrow$ need to implement complex higher order time integration

Discussion: Comparing GNAT with POD-DEIM II

Approximation: $+$

- GNAT is more accurate then POD-DEIM (for the same time discretization), because test space is not reduced, so all equations are used, and the best approximation (in the l^2-norm) is used
- for multiple equations the residual has to be weighted
  $+$ possibility to increase the influence of important equations
  $-$ neccesarity to carefully weight in the present problem
- initial value for the GN iteration is important, if there are multiple local minima,
  (here: state from the last time step / or POD-solution (very good))

Thank you for your attention !