Otto-von-Guericke-Universität Magdeburg
Fakultät für Mathematik – Institut für Analysis und Numerik
Max-Planck-Institut für Dynamik komplexer technischer Systeme
Computational Methods for Systems and Control Theory

Summer Term 2012
Prof. Dr. Peter Benner
Tobias Breiten

Model Reduction for Dynamical Systems – 3. Exercise

Deadline for homework: 05/22/2012 in the exercise

Exercise 1 **(Model reduction by interpolation)**

Given a SISO LTI system

$$\dot{x}(t) = Ax(t) + bu(t),$$
$$y(t) = c^T x(t).$$

Assume that a reduced-order model is given by a Petrov-Galerkin type projection $\mathcal{P} = VW^T$, i.e.

$$\hat{A} = W^T AV,\ \hat{b} = W^T b,\ \hat{c} = V^T c.$$

Show that if $(\sigma I - A)^{-1} b \in \operatorname{range}(V)$ and $(\sigma I - A^T)^{-1} c \in \operatorname{range}(W)$, for $\sigma \in \mathbb{C}\backslash\{\Lambda(A) \cup \Lambda(\hat{A})\}$, the reduced-order transfer function $\hat{H}(s)$ is a Hermite interpolant of $H(s)$ in σ, i.e., it holds

$$H(\sigma) = \hat{H}(\sigma), \quad H'(\sigma) = \hat{H}'(\sigma).$$

Exercise 2 **(Model reduction by modal truncation)**

Consider the following LTI system

$$\dot{x}(t) = \underbrace{\begin{bmatrix} -2.5 & -0.5 & 0 \\ -0.5 & -2.5 & 0 \\ -8 & 4 & -5 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 2 \\ 0 \\ -5 \end{bmatrix}}_{b} u(t),$$

$$y(t) = \underbrace{\begin{bmatrix} 29.5 & -6.5 & 7 \end{bmatrix}}_{c} x(t).$$

Compute the eigenvalue decomposition of the matrix A and subsequently derive the pole-residue expression of the underlying transfer function $H(s) = \sum_{j=1}^{n} \frac{R_j}{s-\lambda_j}$, with λ_j denoting the system poles and $R_j = (cx_j)(y_j)^* b$ denoting the residues. Determine the most dominant pole and construct a corresponding reduced-order model by the method of modal truncation.

Exercise 3 (Homework) **(Stability-preserving model reduction)**

Consider an LTI system of the form

$$\dot{x}(t) = Ax(t) + Bu(t),$$
$$y(t) = Cx(t).$$

Show that it holds:

a) If A is dissipative, i.e., $\Lambda(A + A^T) \subset \mathbb{C}_-$, every reduced-order model obtained by an orthogonal projection $\mathcal{P} = VV^T$ is asymptotically stable.

b) Assume that a reduced-order model is constructed by an oblique projection $\mathcal{P} = VW^T$, where $W = PV(V^TPV)^{-1}$ and P solves

$$A^TP + PA + 2\sigma P < 0.$$

Then the reduced-order model is asymptotically stable with $\mathrm{Re}(\lambda_i(\hat{A})) < -\sigma, \ \forall i$.

Exercise 4 (Homework) **(Minimal balanced realization)**

Consider the following LTI system

$$\dot{x}(t) = \underbrace{\begin{bmatrix} 5 & -7 & 0 & -2 \\ 6 & -8 & 0 & -2 \\ 0 & 0 & -3 & 0 \\ 9 & -9 & 0 & -4 \end{bmatrix}}_{A} x(t) + \underbrace{\begin{bmatrix} 2 \\ 1 \\ 1 \\ 3 \end{bmatrix}}_{b} u(t),$$

$$y(t) = \underbrace{\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}}_{c} x(t).$$

Use the MATLAB command *lyapchol* to compute the Cholesky factors S and R of the solutions to the Lyapunov equations

$$AP + PA^T + bb^T = 0, \quad A^TQ + QA + c^Tc = 0.$$

In case that you do not have access to the Control System toolbox, you can find the results in the file *LyapSol.mat* on the course homepage. Compute the singular value decomposition $U\Sigma V^T = SR^T$ of the product of the Cholesy factors S and R^T. What can you say about the minimality of the system? Modify the procedure from Exercise 1, exercise sheet 2, and suitably adjust the transformation matrices $T = \Sigma^{-\frac{1}{2}}V^TR$ and $T^{-1} = S^TU\Sigma^{-\frac{1}{2}}$. Use your results to construct a minimal reduced-order model by an oblique projection which exactly reproduces the transfer function of the original model, i.e., for the reduced system it should hold $\hat{H}(s) = H(s), \ \forall s$. Validate your results by means of plotting the gain of the original and the reduced transfer function with the values specified in Exercise 5, exercise sheet 2.

Send your routine to breiten@mpi-magdeburg.mpg.de. **The filename should include your name and the corresponding exercise sheet number as well as the exercise number, e.g., name ha3a4. In case of several files please hand in a compressed file. Moreover, please print the source code of your routine and hand it in together with the other exercises.**