

Model Reduction for Dynamical Systems – 4. Exercise

Deadline for homework: 06/05/2012 in the exercise

Exercise 1 (Homework) (Pole-residue expression for second-order systems)

Consider a second-order SISO dynamical system of the form

$$\begin{aligned} M\ddot{x}(t) + D\dot{x}(t) + Kx(t) &= bu(t), \\ y(t) &= c^T x(t). \end{aligned}$$

Show that the transfer function $H(s) = c^T(s^2M + sD + K)^{-1}b$ can be expressed by a generalized pole-residue representation as

$$H(s) = \sum_{i=1}^{2n} \frac{R_i}{s - \lambda_i},$$

with residues R_i of the form

$$R_i = (c^T x_i)(y_i^* b)\lambda_i,$$

and eigentriplets (λ_i, x_i, y_i) , $i = 1, \dots, 2n$, given as

$$\begin{aligned} (\lambda_i^2 M + \lambda_i D + K)x_i &= 0, & \text{right eigenpairs,} \\ y_i^*(\lambda_i^2 M + \lambda_i D + K) &= 0, & \text{left eigenpairs,} \\ -y_i^* K x_i + \lambda_i^2 y_i^* M x_i &= 1, & \text{normalization.} \end{aligned}$$

Exercise 2 (Homework) (Model reduction by balanced truncation)

Implement the method of balanced truncation introduced in the lecture. If you do not have access to the control system toolbox, you can use the routine `lyap_sgn_fac.m` from the course homepage to compute approximations to the Cholesky factors of the solutions to the Lyapunov equations. Try your program by means of the model of a beam which you find as `beam.mat` on the course homepage. Evaluate the transfer function

$$H(i\omega) = C(i\omega I - A)^{-1}B$$

for original and reduced-order model over the frequency interval $\omega \in [10^{-2}, 10^4]$. Use 10 000 logarithmically distributed sample points. Plot the *gain* of the transfer function, i.e. $20 \cdot \log_{10} |(H(j\omega))|$ on a logarithmic x -scale by using the MATLAB command `semilogx(ω , H_ω)`.

Exercise 3 (Homework) (Model reduction by the dominant pole algorithm)

Implement the dominant pole algorithm which is given as follows:

Algorithmus 1 Dominant pole algorithm (DPA)

Input: System (A, b, c) , initial pole s_0 , tolerance $\epsilon \ll 1$.

Output: Approximate dominant eigentriplet (λ, x, y) .

- 1: Set $k = 0$.
- 2: **while** (not converged) **do**
- 3: Solve $(s_k I - A)v_k = b$ for v_k .
- 4: Solve $(s_k I - A)^* w_k = c^*$ for w_k .
- 5: Compute next pole estimate

$$s_{k+1} = s_k - \frac{cv_k}{w_k^* v_k}.$$

- 6: Set $v_k = \frac{v_k}{\|v_k\|_2}$ and $w_k = \frac{w_k}{\|w_k\|_2}$.
 - 7: **if** $\|Av_k - s_{k+1}v_k\|_2 < \epsilon$ **then**
 - 8: Set $\lambda = s_{k+1}$, $x = v_k$, $y = w_k$. **break**
 - 9: **end if**
 - 10: Set $k = k + 1$.
 - 11: **end while**
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Compute reduced-order models of different dimensions and compare your results with the method of balanced truncation by means of the model of the beam specified in Exercise 3. Use the same values as above. What do you observe?

Hint: In order to avoid the recomputation of the same pole again and again, scale the matrices of already found eigenvectors X, Y s.t. $Y^T A X = \Lambda$ and $Y^T X = I$. Then, after each iteration step, replace b and c by the projections $(I - XY^*)b$ and $c(I - XY^*)$, respectively.

Send your routine to breiten@mpi-magdeburg.mpg.de. The filename should include your name and the corresponding exercise sheet number as well as the exercise number, e.g., name ha4a2 and ha4a3, respectively. In case of several files please hand in a compressed file. Moreover, please print the source code of your routine and hand it in together with the other exercises.