Otto-von-Guericke-Universität Magdeburg
Fakultät für Mathematik – Institut für Analysis und Numerik
Max-Planck-Institut für Dynamik komplexer technischer Systeme
Computational Methods for Systems and Control Theory

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Model Reduction for Dynamical Systems – 5. Exercise

Deadline for homework: 12/05/2012 in the exercise

Exercise 1 (Homework) **(Balancing-free square root (BFSR) method)**

Another way of model reduction is to use the balancing-free square root (BFSR) algorithm. Analogue to the standard square-root balanced truncation approach, one has to compute the Cholesky factors S and R of the solutions of the Lyapunov equations and the corresponding SVD of those factors, i.e.,

$$AP + PA^T + BB^T = 0, \quad A^TQ + QA + C^TC = 0, \quad SR^T = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}.$$

The left and right projection matrices for the computation of a reduced-order model of dimension r now are given as $T_l = (Q_1^T P_1)^{-1} Q_1^T$ and $T_r = P_1$, with

$$S^T U_1 = \begin{bmatrix} P_1 & P_2 \end{bmatrix} \begin{bmatrix} \hat{R} \\ 0 \end{bmatrix}, \quad R^T V_1 = \begin{bmatrix} Q_1 & Q_2 \end{bmatrix} \begin{bmatrix} \bar{R} \\ 0 \end{bmatrix},$$

and $P_1, Q_1 \in \mathbb{R}^{n \times r}$ have orthonormal columns and $\hat{R}, \bar{R} \in \mathbb{R}^{r \times r}$ are upper triangular.
Show that the reduced-order system is equivalent to a balanced system and that it satisfies the same error bound as the one obtained by the standard square root BT method.

Exercise 2 (Homework) **(Balanced model reduction for non-minimal systems)**

Consider a system which is neither controllable nor observable, i.e.,

$$\mathcal{K} = \operatorname{rank}\left(\begin{bmatrix} B & AB & \dots & A^{n-1}B \end{bmatrix}\right) = k_1 < n \quad \text{and} \quad \mathcal{O} = \operatorname{rank}\left(\begin{bmatrix} C \\ CA^T \\ \vdots \\ C(A^T)^{n-1} \end{bmatrix}\right) = k_2 < n.$$

Show that if $Y \in \mathbb{R}^{n \times k_1}$ and $Z \in \mathbb{R}^{n \times k_2}$ are low rank factors that satisfy $P = YY^T$ and $Q = ZZ^T$, a balanced reduced-order model can be obtained by projection matrices $T_l = \Sigma_1^{-\frac{1}{2}} V_1 Z^T$ and $T_r = YU_1\Sigma_1^{-\frac{1}{2}}$, where $Y^TZ = \begin{bmatrix} U_1 & U_2 \end{bmatrix} \begin{bmatrix} \Sigma_1 & 0 \\ 0 & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}$.

Exercise 3 (Homework) **(Singular perturbation approximation)**

Implement the singular perturbation approximation algorithm discussed in the lecture. Compute reduced-order models of different dimensions and compare your results with the method of balanced truncation by means of the model of the beam specified in Exercise 3.

Send your routine to breiten@mpi-magdeburg.mpg.de. **The filename should include your name and the corresponding exercise sheet number as well as the exercise number, e.g., name ha5a3, respectively. In case of several files please hand in a compressed file. Moreover, please print the source code of your routine and hand it in together with the other exercises.**