Winterterm 2012/2013

Otto-von-Guericke-University Magdeburg
Max Planck Institute for Dynamics of Complex Technical Systems
Computational Methods for Systems and Control Theory

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Website: http://www.mpi-magdeburg.mpg.de/mpcsc/lehre/2012_WS_SC/

Scientific Computing 1
7th Homework

Handout: 11/15/2012 **Return:** 11/22/2012

Exercise 1: (5 Points)

Compute the forward error for the evaluation of the polynomial

$$P(x) = c_1 x + c_2 x^2$$

a.) using direct evaluation and

b.) using the Horner scheme.

Consider the case where x is close to a root of the polynomial and conclude which of those evaluation techniques is the more stable one. Use this to reinterpret the results of the polynomial evaluation

$$P(x) = x^6 - 998x^5 - 998x^4 - 998x^3 - 998x^2 - 998x - 998$$

at $x = 999$. (See Exercise 4 from from the last homework.)

Exercise 2: (6 Points)

Determine the absolute and the relative condition numbers of

a.) $f(x) = \sin(x)$,

b.) $f(x) = \arctan(x)$,

c.) $f(x) = \sqrt{x \exp(x)}, \quad x > 0.$

Which values of x will lead to high condition numbers?

Exercise 3: (5 Points)

We use the following C code to sum up n real numbers $x_0, \ldots, x_{n-1}$:

```
s = 0;
for (i = 0;  i < n; i++) {
   s += x[i];
}
```

Prove that the resulting forward error can be expressed as

$$\hat{s} - s = \sum_{i=0}^{n-1} \delta_i t_i,$$

where t_i is the exact i-th partial sum and $|\delta_i| < \mathbf{u}$. What is best summation order to minimize $\hat{s} - s$ in the following examples:

a.) $x_i = \frac{1}{i}$,

b.) $\left|\sum\limits_{i=0}^{n-1} x_i\right| \ll \sum\limits_{i=0}^{n-1} |x_i|$,

c.) $x = \left[\mathbf{u}\,\mathbf{u}\,1\,\frac{1}{\mathbf{u}}\ -\frac{1}{\mathbf{u}}\right]$.

Exercise 4: **(5 Points)**

For all $x \in \mathbb{R}^n$ and a fixed $v \in \mathbb{R}^n$ we define the following mapping $f : \mathbb{R}^n \to \mathbb{R}$:

$$f(x) = \langle x, v \rangle = v^T x.$$

Determine the condition of this mapping. For which $x \in \mathbb{R}^n$ is the condition particularly small or particularly large?

Exercise 5: **(4 Points)**

Use a backward error analysis to determine numerical stability of:

a.) $f(x) = ax$ and

b.) $g(x) = a + x$.

Give conditions (where necessary) to guarantee stability. Assume that $a \in \mathbb{R}$ is fixed.

Overall Points: 25