

Scientific Computing 1 5th Homework

Handout: 11/26/2014

Return: 12/03/2014

Exercise 1:

(4 Points)

Compute the forward error for the evaluation of the polynomial

$$P(x) = c_1x + c_2x^2$$

- a.) using direct evaluation and
- b.) using the Horner scheme.

Consider the case where x is close to a root of the polynomial and conclude which of those evaluation techniques is the more stable one.

Exercise 2:

(6 Points)

Determine the absolute and the relative condition numbers of

- a.) $f(x) = \sin(x)$,
- b.) $f(x) = \arctan(x)$,
- c.) $f(x) = \sqrt{x \exp(x)}$, $x > 0$.

Which values of x will lead to high condition numbers?

Exercise 3:

(4 Points)

For all $x \in \mathbb{R}^n$ and a fixed $v \in \mathbb{R}^n$ we define the following mapping $f : \mathbb{R}^n \rightarrow \mathbb{R}$:

$$f(x) = \langle x, v \rangle = v^T x.$$

Determine the condition of this mapping. For which $x \in \mathbb{R}^n$ is the condition particularly small or particularly large?

Exercise 4:**(4 Points)**

Use a backward error analysis to determine numerical stability of:

a.) $f(x) = ax$ and

b.) $g(x) = a + x$.

Give conditions (where necessary) to guarantee stability. Assume that $a \in \mathbb{R}$ is fixed.

Exercise 5:**(7 Points)**

We want to compute the following two integrals

$$I_1 := \int_{-20}^{20} e^x dx$$

and

$$I_2 := \int_{-20}^{20} e^{-x} dx$$

using a C program. Implement the *midpoint rule*:

$$\int_a^b f(x) dx \approx \sum_{i=0}^{n-1} h f\left(a + ih + \frac{1}{2}h\right),$$

where $h := \frac{b-a}{n}$, in **single precision** arithmetic.

The integrals I_1 and I_2 are now approximated by employing $n \in \{2048, 4096, 8192, 16384, 32768\}$ sampling points. Compare the results and prove them by computing the correct value of the integrals using their antiderivatives.

Analyze the reason behind the occurring errors and create a modified version of your implementation which avoids those errors **without** using double precision computations. What happens if the number of sampling points is not a power of 2?

Hint: Use `-O2 -ffloat-store` as compiler flags.

Overall Points: 25