

---

**Scientific Computing 1**  
**Handout 5**  
**November 7, 2016**

---

**Rounding Errors and Error Propagation**

- **Relative rounding errors in  $\mathbb{M}(p, t, e_{\min}, e_{\max})$ :**

$$\frac{|\gamma(x) - x|}{|x|} < \frac{1}{2}p^{1-t} \quad \forall x \in [-x_{\max}, -x_{\min}] \cup [x_{\min}, x_{\max}].$$

- **unit roundoff:**

$$\mathbf{u} := \frac{1}{2}p^{1-t}$$

- **machine epsilon:**

$$\text{eps} := \min\{|\tilde{x} - 1| \mid \tilde{x} \in \mathbb{M}(p, t, e_{\min}, e_{\max}), \tilde{x} > 1\} = p^{1-t} = 2\mathbf{u}$$

- **standard model of the floating point arithmetic:**

$$x \triangleleft y = (x \Delta y)(1 + \delta), \quad |\delta| \leq \mathbf{u} \quad \forall \Delta \in \{+, -, \cdot, /\}$$

(is also expected to hold for  $\sqrt{\phantom{x}}$ ).

- **error propagation:** Let  $\tilde{x} := \gamma(x)$ ,  $\tilde{y} := \gamma(y)$ .

**addition:**  $x, y \in \mathbb{R}$ ,  $\text{sign}(x) = \text{sign}(y) \implies$

$$\frac{|(\tilde{x} \oplus \tilde{y}) - (x + y)|}{|x + y|} \leq 2\mathbf{u} + \mathbf{u}^2$$

**subtraction:**  $x, y \in \mathbb{R}$ ,  $\text{sign}(x) = \text{sign}(y) \implies$

$$\frac{|(\tilde{x} \ominus \tilde{y}) - (x - y)|}{|x - y|} \leq \left(\frac{2|y|}{|x - y|} + 2\right)\mathbf{u} + \left(\frac{2|y|}{|x - y|} + 1\right)\mathbf{u}^2.$$

**cancellation:**  $x \approx y \rightsquigarrow$  large error

**multiplication and division:**

$$\frac{|\tilde{x} \odot \tilde{y} - x \cdot y|}{|x \cdot y|} \leq 3\mathbf{u} + \mathcal{O}(\mathbf{u}^2).$$