Otto von Guericke University Magdeburg
Institute for Analysis and Numerics
Max Planck Institute for Dynamics of Complex Technical Systems
Computational Methods in Systems and Control Theory

Summer Term 2017
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Model Reduction of Dynamical Systems
Homework 3

Deadline: 2 June 2017

Problem 1. *Quadratic model order reduction*

Consider the nonlinear RC circuit example, of size $n = 100$, discussed in the lecture and represent the system in quadratic form. Use a stiff ODE solver to simulate the original nonlinear system and the quadratic form of the system from $t_0 = 0$ to $t_1 = 10$, for the step input u such that $u(t) = 0$ for $t < 3$ and $u(t) = 1$ for $t \geq 3$.

Implement the quadratic MOR method for the RC circuit example with order $r = 20$ using zero as expansion point for the quadratic form. Plot the output of the original nonlinear system, the quadratic form of the nonlinear system, and the reduced quadratic form of nonlinear system with respect to time.

Problem 2. *Carleman bilinearization*

Approximate a nonlinear system

$$\begin{aligned} \dot{x}(t) &= f(x(t)) + Bu(t), \\ y(t) &= Cx(t), \end{aligned}$$

where $x(t) \in \mathbb{R}^n$ and $u(t), y(t) \in \mathbb{R}$, using Carleman bilinearization around x_0, where

$$f(x) \approx f(x_0) + D_f(x_0)(x - x_0) + H_f(x_0)\left((x - x_0) \otimes (x - x_0)\right)$$

with $f(x_0) \neq 0$, $D_f(x_0) \in \mathbb{R}^{n \times n}$, and $H_f(x_0) \in \mathbb{R}^{n \times n^2}$.

Problem 3. *Bilinearization model order reduction*

Consider the nonlinear RC circuit example, of size $n = 100$, discussed in the lecture and apply Carleman bilinearization around $x_0 = 0$. Use a stiff ODE solver to simulate the original nonlinear system and the bilinear form of the system from $t_0 = 0$ to $t_1 = 10$, for the step input u such that $u(t) = 0$ for $t < 3$ and $u(t) = 1$ for $t \geq 3$.

Implement the bilinearization MOR method for the RC circuit example with order $r = 20$, using 19 moments and 1 multi-moment at zero. Plot the output of the original nonlinear system, the bilinear form of the nonlinear system, and the reduced bilinear form of nonlinear system with respect to time.

You can send your solutions in two ways:

1. **by sending them to** `mlinaric@mpi-magdeburg.mpg.de`**,**
2. **by adding Petar Mlinarić (username** `pmli`**) to your GitLab or Bitbucket project.**

Solutions should be written in a PDF file (created using LaTeX) or a Jupyter notebook. When sending emails, please add `[mor17]` **to the subject line.**