Otto-von-Guericke-Universität Magdeburg
Max Planck Institute for Dynamics of Complex Technical Systems
Prof. Dr. Peter Benner, Dr. Patrick Kürschner, Carolin Penke

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Advanced Topics in Numerical Linear Algebra – 4. Exercise

These problems will be discussed in the exercise course of May 31, 2018. Please prepare their solutions before the class, you will be asked to perform one of the questions on the board. You may send your MATLAB® implementations via email to *penke@mpi-magdeburg.mpg.de*.

Problem 1 (Proof of Theorem II.24)

Prove Theorem II.24 for the ARE

$$F + A^T X + XA - XGX = 0, \quad G \geq 0, \quad X = X^T. \tag{1}$$

If $F \geq 0$, then the stabilizing solution X_* of the ARE (1) (if it exists) is positive semi-definite. Furthermore, if (A^T, F) is controllable, then $X_* > 0$.

Problem 2 (ARE solutions from eigenvectors)

Assume $H \in \mathbb{H}_n$ corresponding to (1) is diagonalizable and $\Lambda(H) \cap i\mathbb{R} = \emptyset$. Derive an algorithm for computing the stabilizing solution X_* using the eigenvalue decomposition of H. Implement this method and test it on random A, $F = F^T$, $G = G^T$.

Problem 3 (Symplectic similarity transformations)

Prove Lemma II.26: if $H \in \mathbb{H}_n$ is Hamiltonian and $S \in \mathbb{R}^{2n\times 2n}$ is symplectic, then $\tilde{H} := S^{-1}HS \in \mathbb{H}_n$.

Problem 4 (Bounds for Lyapunov solution)

Let $A \in \mathbb{C}^{n\times n}$ be a Hurwitz matrix and $Q = Q^*$. Assume $\|e^{At}\| \leq e^{-\alpha t}$ $\forall t \geq 0$ with $\alpha > 0$. Show that for the solution of $AX + XA^* + Q = 0$ it holds $\|X\| \leq \frac{\|Q\|}{2\alpha}$.

Problem 5 (Matrix exponential)

Show the following properties of the matrix exponential function.

a) $e^A e^B = e^{A+B}$ for commuting matrices $A,\ B \in \mathbb{C}^{n\times n}$.

b) $e^{A\otimes I_m + I_n \otimes B} = e^A \otimes e^B$ for arbitrary matrices $A \in \mathbb{C}^{n\times n},\ B \in \mathbb{C}^{m\times m}$.

c) $\det\left(e^A\right) = e^{\mathrm{tr}(A)}$.

Hint: User the power series expression $e^A = \sum_{j=0}^{\infty} \frac{A^j}{j!}$.