Otto-von-Guericke-Universität Magdeburg summer term 2018
Max Planck Institute for Dynamics of Complex Technical Systems
Prof. Dr. Peter Benner, Dr. Patrick Kürschner, Carolin Penke

Advanced Topics in Numerical Linear Algebra – 5. Exercise

These problems will be discussed in the exercise course of June 14, 2018. Please prepare their solutions before the class, you will be asked to perform one of the questions on the board. You may send your MATLAB® implementations via email to *penke@mpi-magdeburg.mpg.de*.

Problem 1 (Definitions of $f(A)$)

Consider the matrix

$$A = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

and compute $\sin(\pi A)$ via the JNF of A and also via the Hermite interpolating polynomial of A.

Problem 2 (Equivalence of Definitions)

Let f be defined on $\Lambda(J)$, where $J(\lambda)$ is a Jordan block of size m w.r.t. an eigenvalue $\lambda \in \mathbb{C}$. Use the Definition III.4 of $f(J)$ via the Hermite polynomial to derive the formula $f(J) = \begin{bmatrix} f(\lambda) & f'(\lambda) & \cdots & \frac{f^{(m-1)}(\lambda)}{(m-1)!} \\ & \ddots & \ddots & \vdots \\ & & \ddots & f'(\lambda) \\ 0 & & & f(\lambda) \end{bmatrix}$ from Definition III.2.

Problem 3 (Properties of Matrix Functions)

Let $A \in \mathbb{C}^{n\times n}$ and let f, g be defined on $\Lambda(A)$. Prove the following statements

- **a)** $f(A)$ commutes with A.
- **b)** $f(XAX^{-1}) = Xf(A)X^{-1}$.
- **c)** $f(I_m \otimes A) = I_m \otimes f(A)$.
- **d)** If $h(t) = f(t)g(t)$ then $h(A) = f(A)g(A)$.

Problem 4 (Computing $f(A)$ with Schurform)

Let $A \in \mathbb{C}^{n\times n}$, f be defined on $\Lambda(A)$, and assume (for now) that all n eigenvalues are distinct. Derive a method for computing $f(A)$ via the complex Schurform of A using the properties in **Problem 3**. Implement it in MATLAB and test it with random matrices and functions of your choice.
Hint: You should have no problem finding the diagonals of $f(A)$. For the other entries use property **a)** from Problem **3**.

Problem 5 (Matrix Equations With Matrix Functions)

Let $A \in \mathbb{R}^{n\times n}$ with $\Lambda(A) \subset \mathbb{C}_-$ and $B \in \mathbb{R}^{n\times m}$. Show that

$$X = \int_0^T e^{At} BB^T e^{A^T t}\, dt, \quad T < \infty$$

is the solution of

$$AX + XA^T = -BB^T + e^{AT} BB^T e^{A^T T}.$$