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Summer Term 2019
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Model Reduction of Dynamical Systems
Exercise 4

June 28, 2019

Problem 1. *Model reduction by interpolation*

(a) Consider the LTI, MIMO system given by,

$$\begin{aligned} E\dot{x} &= Ax + Bu, \\ y &= Cx, \end{aligned}$$

where $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$ are the system matrices. Assume the projection matrices $V, W \in \mathbb{R}^{n\times r}$ satisfy $W^TV = V^TW = I_{r\times r}$. Let the corresponding reduced model be,

$$\begin{aligned} \hat{E}\dot{\hat{x}} &= \hat{A}\hat{x} + \hat{B}u, \\ \hat{y} &= \hat{C}x, \end{aligned}$$

Let,

$$\begin{aligned} \text{range}(V) &= \text{span}\Big\{\tilde{B}(s_0), \tilde{A}_B(s_0)\tilde{B}(s_0), \ldots, \tilde{A}_B^{q-1}(s_0)\tilde{B}(s_0)\Big\}, \\ \text{range}(W) &= \text{span}\Big\{\tilde{C}^T(s_0), \tilde{A}_c(s_0)\tilde{C}^T(s_0), \ldots, \tilde{A}_c^{q-1}(s_0)\tilde{C}^T(s_0)\Big\}, \end{aligned}$$

where $\tilde{B}(s_0) = (s_0E - A)^{-1}B$, $\tilde{A}_B(s_0) = (s_0E - A)^{-1}E$, $\tilde{A}_C(s_0) = (s_0E - A)^{-T}E^T$ and $\tilde{C}^T(s_0) = (s_0E - A)^{-T}C^T$.

Assume that the matrix pair (E, A) is regular. Prove that, there are at least 2q moments being equal between original systems and the reduced systems, $M_i = \hat{M}_i, i = 0, 1, \ldots, 2q-1$,that is,

$$C[-(s_0E - A)^{-1}E]^i(s_0E - A)^{-1}B = \hat{C}[-(s_0\hat{E} - \hat{A})^{-1}\hat{E}]^i(s_0\hat{E} - \hat{A})^{-1}\hat{B}, i = 0, 1, \ldots, 2q-1$$

Hint: The following holds true,

$$\begin{aligned} (\tilde{A}_B(s_0))^i\tilde{B}(s_0) &= [(s_0E - A)^{-1}E]^i(s_0E - A)^{-1}B = VW^T[(s_0E - A)^{-1}E]^i(s_0E - A)^{-1}B, \\ (\tilde{A}_C(s_0))^i\tilde{C}^T &= [(s_0E - A)^{-T}E^T]^i(s_0E - A)^{-T}C^T = WV^T[(s_0E - A)^{-T}E^T]^i(s_0E - A)^{-T}C^T \end{aligned}$$

(b) Implement a function for interpolating a SISO LTI system

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu(t), \\ y(t) &= Cx(t), \end{aligned}$$

at specified interpolation points, assuming the interpolation points are pairwise distinct and closed under conjugation. Use projection-based approach, i.e. project using matrices V and W such that

$$\text{range}(V) = \text{span}\big\{(\sigma_1 I - A)^{-1}B, \ldots, (\sigma_r I - A)^{-1}B\big\}$$

and

$$\text{range}(W) = \text{span}\{(\sigma_1 I - A)^{-T}C^T, \dots, (\sigma_r I - A)^{-T}C^T\},$$

where $\sigma_1, \dots, \sigma_r$ are the interpolation points. Ensure that V and W are real matrices. Test your function on the clamped beam model `beam.mat` [1].

Problem 2. *Iterative rational Krylov algorithm*
Implement the iterative rational Krylov algorithm (IRKA) discussed in the course for SISO LTI systems. Use the clamped beam model `beam.mat` [1] for testing. Compare the transfer functions of the original and the reduced-order Hermite interpolant and compute the relative $\mathcal{H}_2$-error for different reduced orders.

References

[1] http://slicot.org/20-site/126-benchmark-examples-for-model-reduction.