Model Reduction of Dynamical Systems

Exercise 5

July 05, 2019

Problem 1. *Proper Orthogonal Decomposition (POD) method*

Apply the POD method to the Burgers’ equation described in the domain $x \in [0, 1]$ and time $t \in [0, 10]$ by,

$$\frac{\partial u(x, t)}{\partial t} = \mu \frac{\partial^2 u(x, t)}{\partial x^2} - \frac{\partial}{\partial x} \left( \frac{u(x, t)^2}{2} \right),$$

$u(x, 0) = 0,$

$u(0, t) = \cos(2\pi t), \quad \frac{\partial u(1, t)}{\partial x} = 0$

Use the script given in the coursepage to get the discretized system matrices and the nonlinear function. You can use, for example, the `ode15s` solver in MATLAB to obtain the solution. Compare the trajectories of the original and reduced model.

Problem 2. *POD with DEIM*

Implement the POD with the Discrete Empirical Interpolation Method (DEIM) in order to reduce the computational complexity of the nonlinear function in the Burgers’ equation example. Plot the output of the original, the POD reduced model, and the POD-DEIM reduced model and compare the results.