Otto-von-Guericke-University Magdeburg
Max Planck Institute for Dynamics of Complex Technical Systems
Computational Methods in Systems and Control Theory

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Website: http://www.mpi-magdeburg.mpg.de/csc/teaching/20ws/sc1/

Winterterm 2020/2021

Scientific Computing 1

4th worksheet for online events

12/17/2020

Exercise 1:

Let V be an n-dimensional pre-Hilbert space over $\mathbb{R}$. The inner product is defined as

$$\langle x, y\rangle := x^T y.$$

Show that this is a proper definition of an inner product and prove that

$$||x|| = \sqrt{\langle x, x\rangle}$$

defines a norm on this space.

Exercise 2:

Let $x \in \mathbb{R}^n$. Prove the following inequalities:

a.) $||x||_2 \leq ||x||_1 \leq \sqrt{n}||x||_2$

b.) $||x||_\infty \leq ||x||_2 \leq \sqrt{n}||x||_\infty$

c.) $\frac{1}{n}||x||_1 \leq ||x||_\infty \leq ||x||_1$

Use the results to show the corresponding inequalities for a matrix $A \in \mathbb{C}^{n\times n}$. Finally, show that all norms are finite.

Exercise 3:

Prove that the linear system $Ax = b$ is solvable if and only if

$$\operatorname{rank}(A) = \operatorname{rank}([A\ b])$$

Does this condition guarantee uniqueness?

Exercise 4:

Given the matrices $A, B \in \mathbb{R}^{5\times 5}$

$$A = \begin{bmatrix} 1.0 & 0.0 & 0.5 & 0.0 & -0.5 \\ 0.0 & 2.0 & 0.0 & 0.0 & 0.0 \\ 0.5 & 0.0 & 3.0 & 0.1 & 0.25 \\ 0.0 & 0.0 & 0.1 & 4.0 & 0.0 \\ -0.5 & 0.0 & 0.25 & 0.0 & 5.0 \end{bmatrix}, \qquad B = \begin{bmatrix} 1.0 & 0.0 & 0.5 & 0.0 & -0.5 \\ 0.0 & 2.0 & 0.0 & 0.0 & 0.0 \\ -0.25 & 0.0 & 3.0 & 0.1 & 0.25 \\ 0.0 & 0.0 & 0.1 & 4.0 & 0.0 \\ 0.75 & 0.0 & 0.1 & 0.0 & 5.0 \end{bmatrix},$$

show how they would be represented as:

a.) CSC,

b.) CSR,

c.) ELL,

d.) ELLR.

Further, write matrix-market storage files[1] for both of them.

[1] https://math.nist.gov/MatrixMarket/reports/MMformat.ps