Otto-von-Guericke-University Magdeburg
Max Planck Institute for Dynamics of Complex Technical Systems
Computational Methods in Systems and Control Theory

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Website: http://www.mpi-magdeburg.mpg.de/csc/teaching/20ws/sc1/

Scientific Computing 1

7th worksheet for online events

01/11/2021

Exercise 1:

The Jacobi method is a classical splitting technique to solve linear systems iteratively. It splits a matrix $A \in \mathbb{R}^{n \times n}$ into the diagonal $D = \operatorname{diag}\{a_{11}, a_{22}, \ldots, a_{nn}\}$ and off-diagonal elements and repeatedly solves the equation

$$x_{i+1} = M_J x_i + D^{-1} b$$

where the iteration matrix is

$$M_J = D^{-1}(D - A).$$

Show that the Jacobi method converges to the solution $x = A^{-1}b$ if the matrix A is strictly diagonal-dominant, i.e.,

$$\sum_{j=1,\ j \neq i}^{n} |a_{ij}| < |a_{ii}|, \qquad \forall i = 1, \ldots, n.$$

Hint: Use Theorem 6.25, or Theorem 6.28.

Exercise 2:

Consider the full-rank matrices $A \in \mathbb{R}^{n \times n}$, $V_m \in \mathbb{R}^{n \times m}$ and $W_m \in \mathbb{R}^{n \times m}$.

a.) Prove that

$$P := I - AV_m(W_m^H AV_m)^{-1} W_m^H$$

defines a projection.

b.) Show that

$$PAV_m = 0$$

holds.

Exercise 3:

The sparse matrix vector product is on of the dominant operations in a Krylov subspace method. Consider two matrices $A, \tilde{A} \in \mathbb{R}^{n \times n}$ with $\mathtt{nnz}(A) = \mathtt{nnz}(\tilde{A})$. A has all its nonzero elements in a narrow band along the diagonal, while $\tilde{A}$ has them all over the place. Are matrix vector products

- faster with A,
- faster with $\tilde{A}$,
- equally fast?

Why do you think so?

Exercise 4:

Let $A \in \mathbb{R}^{n \times n}$ symmetric and positive definite. Show that

$$(x, y)_A := (Ax, y)_2$$

defines an inner product and thus

$$\|x\|_a := \sqrt{(x, x)_A}$$

is a norm on $\mathbb{R}^{n \times n}$. (Compare Remark 8.11)

Exercise 5:

Give arguments why the CG method is more desirable compared to splitting methods.
What could be drawbacks of general Krylov subspace methods in this comparison?

Exercise 6:

Can the vector operations in CG be implemented using BLAS routines?