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Please send your solutions (including a MATLAB®implementation if applicable) by **Oct 24, 11:59 PM (Magdeburg students), Oct 18, 11:59 PM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject `NLA-HW01`. Late submissions are only possible if requested by email before the due date for a valid reason.

Numerical Linear Algebra – homework #01

Problem 1 (Inner product)

Show that for $S \in \mathbb{R}^{n\times n}$ the mapping

$$\langle\cdot,\cdot\rangle_S : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}, \qquad (x,y) \mapsto \langle x,y\rangle_S := x^\top S y$$

is an inner product if and only if S is symmetric positive definite.

Problem 2 (Normal matrices)

Prove that for a matrix $A \in \mathbb{C}^{n\times n}$ the following properties are equivalent:

(i) A is normal, i.e. $AA^{\mathrm{H}} = A^{\mathrm{H}}A$.

(ii) $U^{\mathrm{H}}AU$ is normal for all unitary matrices $U \in \mathbb{C}^{n\times n}$.

(iii) For all unitary matrices $U \in \mathbb{C}^{n\times n}$ with $U^{\mathrm{H}}AU = B = \begin{bmatrix} B_{11} & B_{12} \\ 0 & B_{22} \end{bmatrix}$, $B_{11} \in \mathbb{C}^{k\times k}$ it holds $B_{12} = 0$

Hint. Use the trace operator.

(iv) There exists a unitary matrix $U \in \mathbb{C}^{n\times n}$ such that $U^{\mathrm{H}}AU$ is diagonal.

Hint. Use the Schur decomposition.

(v) There exists a unitary matrix $U \in \mathbb{C}^{n\times n}$ such that $A^{\mathrm{H}} = UA$.

(vi) $(Ax, Ay) = (A^{\mathrm{H}}x, A^{\mathrm{H}}y)$ for all $x,\ y \in \mathbb{C}^n$.