Otto-von-Guericke-Universität Magdeburg winter semester 2021/2022
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Please send your solutions (including a MATLAB® implementation if applicable) by **Nov 7, 11:59 PM (Magdeburg students), Nov 1, 11:59 PM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject `NLA-HW03`. Late submissions are only possible if requested by email before the due date for a valid reason.

# Numerical Linear Algebra – homework #03

### Problem 1 (Sherman–Morrison–Woodbury formula)

Let $A \in \mathbb{R}^{n\times n}$ be non-singular, $U$, $V \in \mathbb{R}^{n\times k}$ for $k \leq n$ (often it is $k \ll n$). Let $I_k \in \mathbb{R}^{k\times k}$ denotes the identity matrix.

**a)** Prove the Sherman–Morrison–Woodbury formula

$$(A + UV^{\mathrm{T}})^{-1} = A^{-1} - A^{-1}U(I_k + V^{\mathrm{T}}A^{-1}U)^{-1}V^{\mathrm{T}}A^{-1}$$

provided $(I_k + V^{\mathrm{T}}A^{-1}U) \in \mathbb{R}^{k\times k}$ is non-singular.

**b)** Write down an algorithm that solves for given $A$, $U$, $V$, $b$ the linear equation

$$(A + UV^{\mathrm{T}})x = b$$

without computing the inverse $(A + UV^{\mathrm{T}})^{-1}$ explicitly. Assume therefore that a method for solving equations $Ay = f$ is readily available. Discuss advantages and disadvantages compared to direct methods.

### Problem 2 (Classic splitting methods)

We consider the splitting $A = M - N$ with $M$ non-singular, and the stationary iteration techniques from the lecture. Implement in MATLAB:

- Richardson Iteration: $M = \frac{I}{\alpha}$, $N = (-A + \frac{I}{\alpha})$, $\alpha > 0$,
- Jacobi Iteration: $M = D$, $N = L + U$,
- Gauss–Seidel Iteration: $M = D - L$, $N = U$,

where $A = D - L - U$ (see lecture). Test your implementation for the matrix `A=delsq(numgrid('S',n));` (5-point stencil Poisson matrix on $[0, 1]^2]$) for $n = 30$ and the right side `b=sum(A,2)`. To illustate the results, you can use the residual norms $\|r_k\|_2$, $k = 1, 2, \ldots$ and plot them.