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Please send your solutions (including a MATLAB®implementation if applicable) by **Nov 14, 11:59 PM (Magdeburg students), Nov 15, 11:59 PM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject `NLA-HW04`. Late submissions are only possible if requested by email before the due date for a valid reason.

Numerical Linear Algebra – homework #04

Problem 1 (Gauss Seidel Relaxation)

Let $A \in \mathbb{C}^{n\times n}$ with $a_{ii} \neq 0$ for $i = 1, \dots, n$. Assume additionally that $A = D + L + R$, where D is a diagonal matrix, L the part below the diagonal and R the part above the diagonal. Show that for $\omega \in \mathbb{R}$ it holds

$$\rho((D + \omega L)^{-1}(D + \omega L - \omega A)) \geq |w - 1|.$$

Problem 2 (Chebyshev polynomials)

a) Prove for the Chebyshev polynomials τ_k, $k = 0, 1, 2, \dots$, which are defined by the recursion

$$\begin{aligned} \tau_0(t) &= 1, \\ \tau_1(t) &= t, \\ \tau_{k+1}(t) &= 2t\tau_k(t) - \tau_{k-1}(t), \end{aligned}$$

that the following explicit representation holds true:

$$\tau_k(t) = \tfrac{1}{2}\left[\left(t + \sqrt{t^2 - 1}\right)^k + \left(t - \sqrt{t^2 - 1}\right)^k\right].$$

b) Consider the space of continuous functions on $[-1, 1]$ equipped with the inner product

$$\langle f, g\rangle := \int_{-1}^{1} f(t)\, g(t)\, \frac{\mathrm{d}t}{\sqrt{1 - t^2}}$$

and the subordinate norm $\|f\| = \langle f, f\rangle^{\frac{1}{2}}$. Show that the polynomials τ_k, $k = 0, 1, \dots$, are orthogonal in this space.