

Chemnitz University of Technology
Faculty of Mathematics
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Please send your solutions (including a MATLAB[®] implementation if applicable) by **Nov 21, 11:59 PM (Magdeburg students)**, **Nov 22, 11:59 PM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject NLA-HW05. Late submissions are only possible if requested by email before the due date for a valid reason.

Numerical Linear Algebra – homework #05

Problem 1 (Krylov spaces)

Let $A \in \mathbb{R}^{n \times n}$ and $q_1 \in \mathbb{R}^n$. Show that the following statements are equivalent:

- a) $\{q_1, \dots, A^{k-1}q_1\}$ are linearly independent.
- b) $\mathcal{K}_{k-1}(A, q_1) = \mathcal{K}_{k+\ell}(A, q_1), \forall \ell \geq 0$.
- c) $\mathcal{K}_{k-1}(A, q_1)$ is an A -invariant subspace.

Problem 2 (Steepest Descent & CG)

Implement the method of the steepest descent and the CG method. Compare both methods for the matrices from homework 3 $A = \text{delsq}(\text{numgrid}('S', n))$ (Poisson-Matrix bzgl. $[0; 1]^2$) for $n = 30$ and the right-hand side $b = \text{sum}(A, 2)$. Compare the 2-norm of the residual, and the error norms $\|x - x_k\|_2$ and $\|x - x_k\|_A$ and plot the results.