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Please send your solutions (including a MATLAB®implementation if applicable) by **Dec 12, 11:59 PM (Magdeburg students), Dec 13, 11:59 PM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject `NLA-HW08`. Late submissions are only possible if requested by email before the due date for a valid reason.

Numerical Linear Algebra – homework #08

Problem 1 (Preconditioned Iterations)

Let $A,\ P \in \mathbb{R}^{m\times n}$ be symmetric, positive definite matrices.

a) Show that P^{-1} is symmetric, positive definite in the P-inner product $(\cdot,\cdot)$.

b) Show that AP^{-1} is symmetric in the P^{-1}-inner product $(\cdot,\cdot)$.

c) Show for $A = M - N$ symmetric, positive definite and $M = M^{\mathrm{T}}$ that $G = I - M^{-1}A$ is symmetric in the A-inner product.

d) Derive a right sided-preconditioned CG-method using the statements above.

Problem 2 (Eigenvalues and eigenvectors)

Determine the eigenvectors and eigenvalues of the following 6 matrices:

a) $A = \begin{bmatrix} -5 & 6 \\ -4 & 6 \end{bmatrix}$,

b) $B = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}$,

c) $C = \begin{bmatrix} 15 & 9 \\ -16 & -9 \end{bmatrix}$,

d) $D = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & -4 \\ -2 & 1 & -2 \end{bmatrix}$,

e) $E = \begin{bmatrix} 3 & -10 & -10 \\ 0 & 3 & 0 \\ 0 & -5 & -2 \end{bmatrix}$,

f) $F = \begin{bmatrix} 2 & 2 & 1 \\ 0 & 2 & 1 \\ 0 & -1 & 0 \end{bmatrix}$.