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Please send your solutions (including a MATLAB®implementation if applicable) by **Dec 19, 11:59 PM (Magdeburg students), Dec 20, 11:59 PM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject `NLA-HW09`. Late submissions are only possible if requested by email before the due date for a valid reason.

Numerical Linear Algebra – homework #09

Problem 1 **(Perturbation theory)**

Let $Q^*AQ = D + N$ be a Schur decomposition of $A \in \mathbb{C}^{n\times n}$, i.e.

$$N = \begin{bmatrix} 0 & & \ast \\ & \ddots & \\ & & 0 \end{bmatrix}, \qquad D = \begin{bmatrix} \lambda_1 & & \\ & \ddots & \\ & & \lambda_n \end{bmatrix}, \qquad \Lambda(A) = \{\lambda_1, \dots, \lambda_n\}$$

and $Q \in \mathbb{C}^{n\times n}$ be unitary. Then for $\mu \in \Lambda(A+E)$ it holds:

$$\min_{\lambda\in\Lambda(A)} |\lambda - \mu| \le \max\left\{\Theta, \Theta^{\frac{1}{p}}\right\},$$

where

$$\Theta = \|E\|_2 \sum_{k=0}^{p-1} \|N\|_2^k$$

and p is the nilpotency index of N, i.e. $p = \min\{q \in \mathbb{N}_0 \mid N^q = 0\}$.
Hint: Compare with the proof from Bauer-Fike; you need the Neumann series.

Problem 2 **(Perturbation in the eigenvector)**

Let $A = \begin{bmatrix} 1 & \epsilon & \epsilon \\ \epsilon & 1 & \epsilon \\ \epsilon & \epsilon & 1 \end{bmatrix}$ with $\epsilon \in \mathbb{R}$

a) What is the eigenvalue corresponding to the eigenvector $(1,1,1)^T$.

b) Determine the eigenvalues and eigenvectors of the matrix $A - B$ with $B = \begin{bmatrix} 0 & 0 & \epsilon \\ 0 & 0 & 0 \\ \epsilon & 0 & 0 \end{bmatrix}$.

c) Compute $\alpha = \min\left\{\frac{\|y-x\|_2}{\|x\|_2} : x = (1,1,1)^T, y \text{ Eigenvektor von } A - B\right\}$, the perturbation in the eigenvectors. Compare those with the perturbation in the eigenvalue.