

Chemnitz University of Technology
Faculty of Mathematics
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Please send your solutions (including a MATLAB[®] implementation if applicable) by **Jan 16, 11:59 PM (Magdeburg students)**, **Jan 17, 11:59 PM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject **NLA-HW10**. Late submissions are only possible if requested by email before the due date for a valid reason.

Numerical Linear Algebra – homework #10

Problem 1 (Improvement of QR iteration)

We know that the classic QR iteration determines a Schur decomposition of a matrix A if it converges.

Algorithm 1 OR iteration

```
1:  $H_0 := Q_0^* A Q_0$ , ( $Q_0 = I$ )
2: for  $j = 1, 2, \dots$  do
3:    $H_{k-1} =: QR$ 
4:    $H_k = RQ$ 
5: end for
```

- Implement a classic QR iteration. Think about a useful stopping criterion.
- Now add a Hessenberg transformation in the beginning of your method. That means that Q_0 is now the corresponding matrix that transforms A into a Hessenberg form.
- Now add a simple shift $\mu_k = h_{n,n}$ to the iteration as well as the usage of deflation.

Compare the runtimes for all three methods for the matrices $A1 = \text{wilkinson}(10)$, $A2 = \text{rand}(10)$. Explain your observations.

Hint: You can compute the Hessenberg form of matrix using the Matlab function `hess(A)`.