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Please send your solutions (including a MATLAB® implementation if applicable) by **Jan 30 (Magdeburg students), Jan 28, 11:30 AM (Chemnitz students)** to przybilla@mpi-magdeburg.mpg.de (Magdeburg students) or jan.blechta@math.tu-chemnitz.de (Chemnitz students) with subject `NLA-HW12`. Late submissions are only possible if requested by email before the due date for a valid reason.

Numerical Linear Algebra – homework #12

Problem 1 (Part b) of Theorem from page 14/18)

Proof the following statement: Let $A = A^T \in \mathbb{R}^{n\times n}$ with $Ax_j = \lambda_j x_j$ where $\lambda_1 \geq ... \geq \lambda_n$ and $x_j^T x_k = \delta_{jk}$. Moreover, let $T_k = T_k^T$ be the tridiagonal matrix after k steps of the Lanczos algorithm. Then it holds

$$\lambda_n < \Theta_k < \lambda_n + \frac{(\lambda_1 - \lambda_n)\tan^2\phi_n}{(t_{k-1}(1+2\rho_n))^2}$$

where $\rho_n := \frac{\lambda_{n-1}-\lambda_n}{\lambda_1-\lambda_{n-1}}$ and $\phi_n := \arccos|q_1^T x_n|$ and $t_{k-1}(x)$ is the Tchebycheff polynomial of degree $k-1$.