Optimal Control-Based Feedback Stabilization of Multi-Field Flow Problems

Eberhard Bänsch[1] Peter Benner[2,3] Jens Saak[2,3] Martin Stoll[2]
Heiko K. Weichelt[3] Stephan Weller[1]

[1] Friedrich-Alexander-Universität Erlangen-Nürnberg
Department of Applied Mathematics III

[2] Max Planck Institute for Dynamics of Complex Technical Systems Magdeburg,
Research group Computational Methods in Systems and Control Theory

[3] Chemnitz University of Technology, Department of Mathematics,
Research group Mathematics in Industry and Technology

SPP1253 Final Meeting
Banz, Februar 24-26 2013

Overview

Project Summary

Basics/Goals

Basics

- Consider incompressible, instationary Navier-Stokes equations (NSE).
- Riccati-based feedback stabilization with boundary control input.
- Analytical approach by [RAYMOND since 2005].
- Ideas for numerical treatment based on [BÄNSCH/BENNER '10].

Goals

- Find discrete version of *Leray* projection.
- Stabilization using LQR restricted to space of divergence free functions.
- Adapt Newton-ADI algorithms to solve projected LQR problems.
- Apply feedback in forward simulations using NAVIER.
- Extend approach to coupled multi-field flow problems.

Scenarios

Scenario 0: NSE on von Kármán Vortex Street

PDE: NSE

Goal: $\vec{z} = \vec{v} - \vec{w} \to 0$

$\rightsquigarrow$ Linearized Navier-Stokes equations:

$$\frac{\partial \vec{z}}{\partial t} - \frac{1}{\mathrm{Re}}\Delta \vec{z} + (\vec{z}\cdot\nabla)\vec{w} + (\vec{w}\cdot\nabla)\vec{z} + \nabla p = 0$$

$$\mathrm{div}\,\vec{z} = 0$$

defined for $t \in (0,\infty)$ and $\vec{x} \in \Omega$

+ boundary and initial conditions

LQR

Minimize

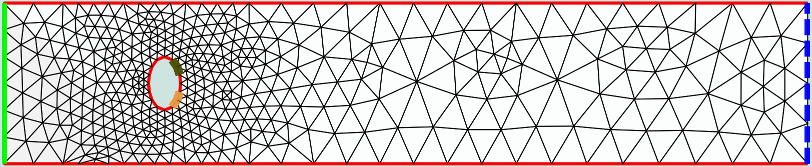
$$\mathcal{J}(\mathbf{y},\mathbf{u}) = \frac{1}{2}\int_0^\infty \lambda||\mathbf{y}||^2 + \frac{1}{\rho}||\mathbf{u}||^2 \,\mathrm{dt}$$

s.t.

$$\begin{bmatrix} M & 0 \\ 0 & 0 \end{bmatrix} \frac{d}{dt}\begin{bmatrix} \mathbf{z} \\ \mathbf{p} \end{bmatrix} = \begin{bmatrix} A & G \\ G^T & 0 \end{bmatrix}\begin{bmatrix} \mathbf{z} \\ \mathbf{p} \end{bmatrix} + \begin{bmatrix} B \\ 0 \end{bmatrix}\mathbf{u}$$

$$\mathbf{y}(t) = C\mathbf{z}(t)$$

Domain Ω: *von Kármán vortex street*



Scenarios

Scenario 0: NSE on von Kármán Vortex Street

NSE

stationary NSE

PDE: NSE

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+ boundary and initial conditions

LQR

Minimize

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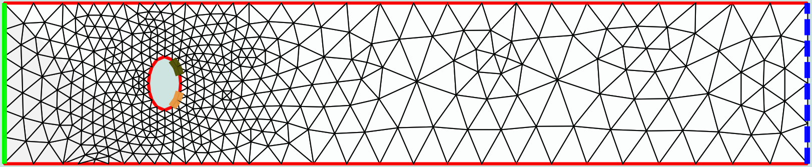

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$+$ boundary and initial conditions

LQR

Minimize

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s.t.

$$\mathcal{M} \frac{d}{dt} \tilde{\mathbf{z}} = \mathcal{A} \tilde{\mathbf{z}} + \mathcal{B} \mathbf{u}$$

$$\mathbf{y}(t) = \mathcal{C} \tilde{\mathbf{z}}$$

[Heinkenschloss/Sorensen/Sun '08]

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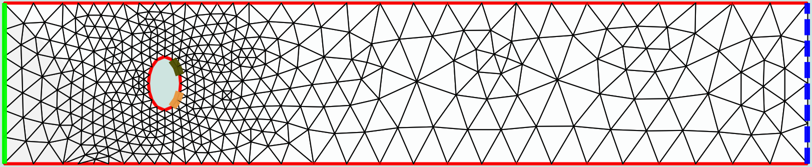


Scenarios

Scenario 1: NSE Coupled with DCE in Reactor Model

PDE: NSE+DCE

Goal: $\vec{z} = \vec{v} - \vec{w} \to 0, c = c^{(\vec{v})} - c^{(\vec{w})} \to 0$

$\rightsquigarrow$ Linearized coupled system:

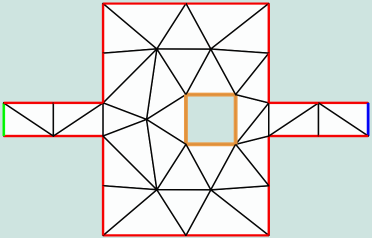
$$\frac{\partial \vec{z}}{\partial t} - \frac{1}{\mathrm{Re}}\Delta \vec{z} + (\vec{z}\cdot\nabla)\vec{w} + (\vec{w}\cdot\nabla)\vec{z} + \nabla p = 0$$

$$\frac{\partial c}{\partial t} - \frac{1}{\mathrm{Re}\,\mathrm{Sc}}\Delta c + (\vec{w}\cdot\nabla)c + (\vec{z}\cdot\nabla)c^{(\vec{w})} = 0$$

$$\operatorname{div}\vec{z} = 0$$

defined for $t \in (0,\infty), \vec{x} \in \Omega$ and some BC,IC

Domain Ω: Reactor Model



LQR

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s.t.

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DCE

stationary DCE

PDE: NSE+DCE

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$$\frac{\partial c}{\partial t} - \frac{1}{\mathrm{Re}\,\mathrm{Sc}}\Delta c + (\vec{w}\cdot\nabla)c + (\vec{z}\cdot\nabla)c^{(\vec{w})} = 0$$

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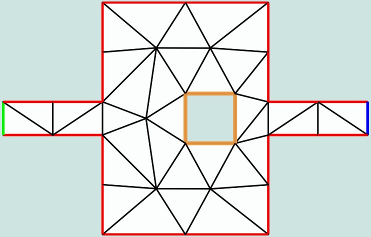

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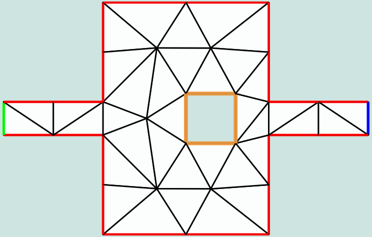

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$$\mathbf{y}(t) = \mathcal{C}\tilde{\mathbf{c}}$$

[Heinkenschloss/Sorensen/Sun '08]

Workflow

LQR for Nonlinear PDEs with Algebraic Constraints

Continuous Level

- Linearize around a given stationary trajectory.
- Index reduction via projection method. [Heinkenschloss/Sorensen/Sun '08]
- Formulate stabilization problem for the perturbation.

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Workflow

Nested Iteration

Compute feedback matrix $\mathcal{K} = \mathcal{B}^T X \mathcal{M}$ with X solves:

$$\mathcal{R}(X) = \mathcal{C}^T\mathcal{C} + \mathcal{A}^T X\mathcal{M} + \mathcal{M}^T X\mathcal{A} - \mathcal{M}^T X\mathcal{B}\mathcal{B}^T X\mathcal{M} = 0$$

Newton Kleinman method

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Step $\mathbf{m+1}$: solve Lyapunov equation

$$(\mathcal{A} - \mathcal{B}\mathcal{K}^{(m)})^T X^{(m+1)} \mathcal{M} + \mathcal{M}^T X^{(m+1)} (\mathcal{A} - \mathcal{B}\mathcal{K}^{(m)}) = -(\mathcal{W}^{(m)})^T \mathcal{W}^{(m)}$$

Newton Kleinman method

low rank ADI method

Workflow

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Step i: solve the projected linear system

$$(\mathcal{A} - \mathcal{B}\mathcal{K}^{(m)} + q_i \mathcal{M})^T \mathcal{V}_i = \mathcal{Y} \qquad (1)$$

Newton Kleinman method

low rank ADI method

Krylov solver

Workflow

Nested Iteration

Compute feedback matrix $\mathcal{K} = \mathcal{B}^T X \mathcal{M}$ with X solves:

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Krylov solver

Avoid explicit projection using [Heinkenschloss/Sorensen/Sun '08]:

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Krylov solver

Avoid explicit projection using [Heinkenschloss/Sorensen/Sun '08]:
Replace (1) and **solve instead** the saddle point system (SPS)

$$\begin{bmatrix} A^T - (K^{(m)})^T B^T + q_i M^T & G \\ G^T & 0 \end{bmatrix} \begin{bmatrix} V_i \\ * \end{bmatrix} = \begin{bmatrix} Y \\ 0 \end{bmatrix}$$

for different ADI shifts q_i for a couple of rhs Y.

Workflow
Additional Tasks

- Compute initial feedback for unstable systems.
  - ↪ Determine the invariant unstable subspace $\mathcal{U}$.
  - ↪ Solve Bernoulli equation on $\mathcal{U}$ [Benner '11, Amodei/Buchot '12].

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- Derive an efficient variant of large-scale Newton-ADI.
  - ↪ Preprint SPP1253-090 [Benner/Saak '10].

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- Calculate ADI shift parameters depending on the problem structure.
  - $\hookrightarrow$ Different methods have been tested (Penzl, Wachspress, Saak).
  - $\hookrightarrow$ Infinite eigenvalues of DAE pencil yield additional difficulties.

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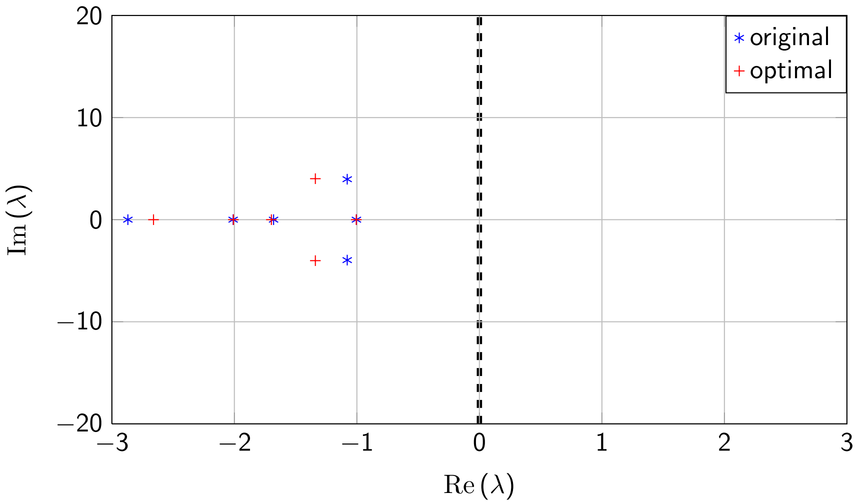
- Preconditioned iterative solvers for innermost saddle point systems.
  - ↪ Preprint SPP1253-130 for Stokes equations [Benner/Saak/Stoll/W. '12].
- Parameter influence observation during the nested iteration.
  - ↪ 3 stopping criteria, Reynolds & Schmidt number, ADI shifts, regularization parameters in cost functional.

Numerical Examples

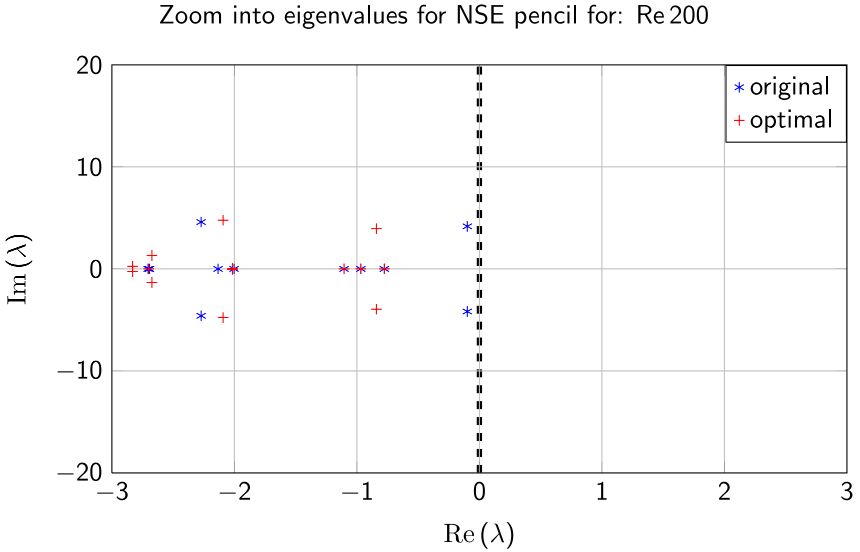
Eigenvalue Behavior of NSE on von Kármán Vortex Street

Zoom into eigenvalues for NSE pencil for: Re 100



Numerical Examples

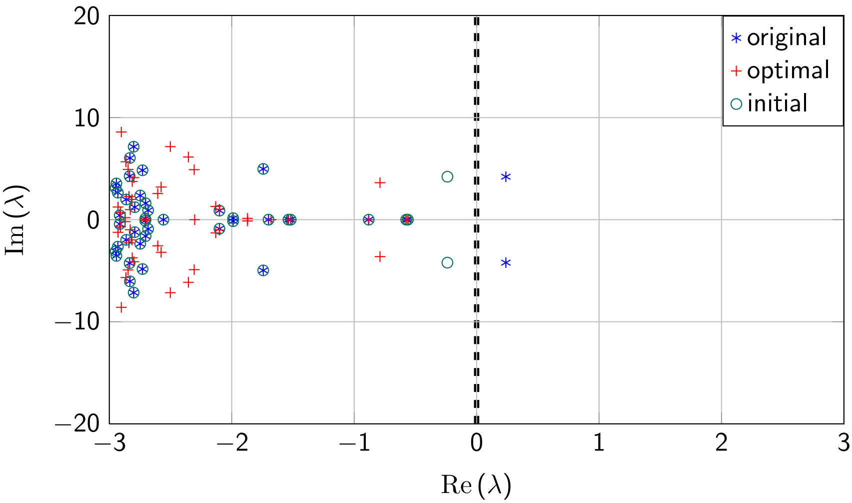
Eigenvalue Behavior of NSE on von Kármán Vortex Street



Numerical Examples

Eigenvalue Behavior of NSE on von Kármán Vortex Street

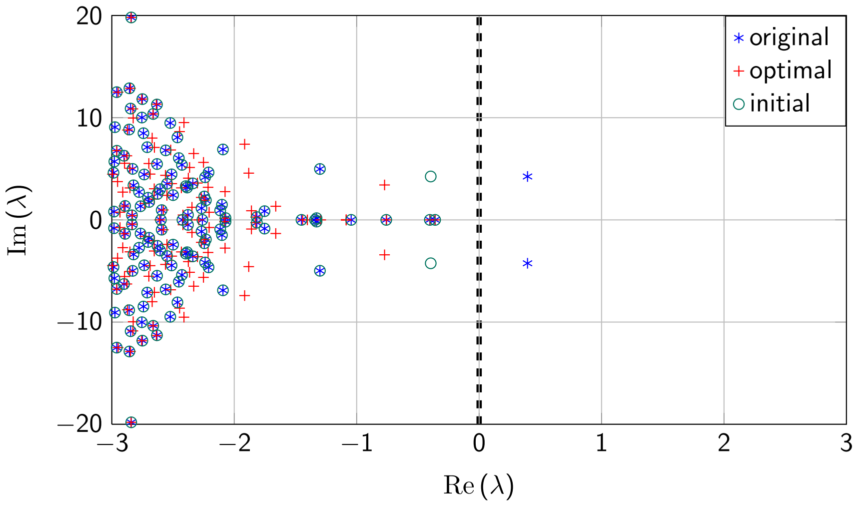
Zoom into eigenvalues for NSE pencil for: Re 300



Numerical Examples

Eigenvalue Behavior of NSE on von Kármán Vortex Street

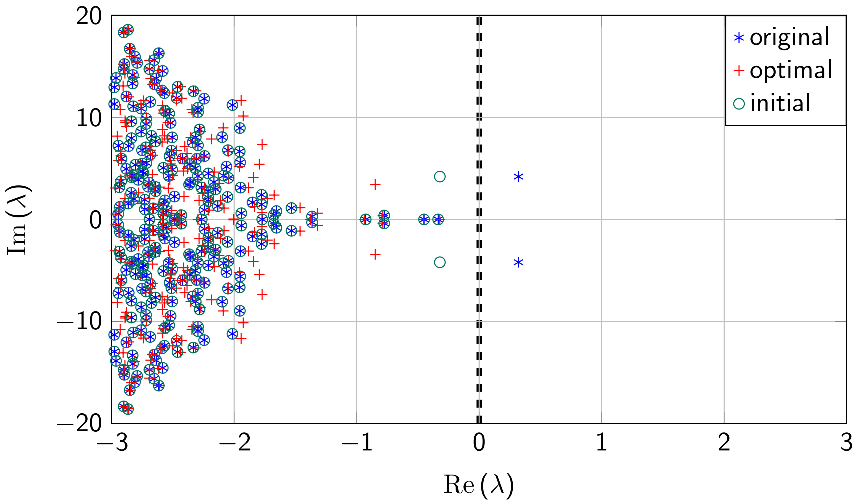
Zoom into eigenvalues for NSE pencil for: Re 400



Numerical Examples

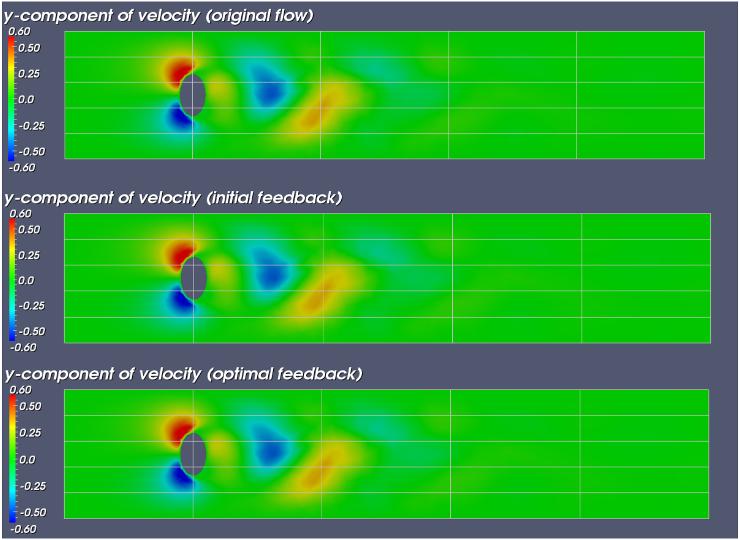
Eigenvalue Behavior of NSE on von Kármán Vortex Street

Zoom into eigenvalues for NSE pencil for: Re 500



Numerical Examples

Closed-Loop Simulation of NSE on von Kármán Vortex Street for Re = 300



Numerical Examples

Closed-Loop Simulation of NSE on von Kármán Vortex Street for Re = 300

Conclusion

Review

- Numerical concept has been implemented.
  - Adapt Newton-ADI algorithm and ADI shift determination for flow problems (DAE structure).
  - Identify initial feedback via Bernoulli equation.
  - Derived reasonable preconditioner to solve SPS iteratively.
- Scenario 0 fully processed and integrated in NAVIER.
- Scenario 1 fully processed without visualization.
- Non-conforming finite elements that guarantee $\operatorname{div} \vec{v} = 0$.

  $\hookrightarrow$ MPI Preprint: [Benner/Saak/Schieweck/Skrzypacz/W. '12]

Outlook

- Visualization of Scenario 1.
- Derive LQR setting for Scenario 2 and 3.
- Adapt methods to special structure of Scenarios 2 and 3.
- Extend new non-conforming composed FE concept.

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Many thanks for your attention!

Literature

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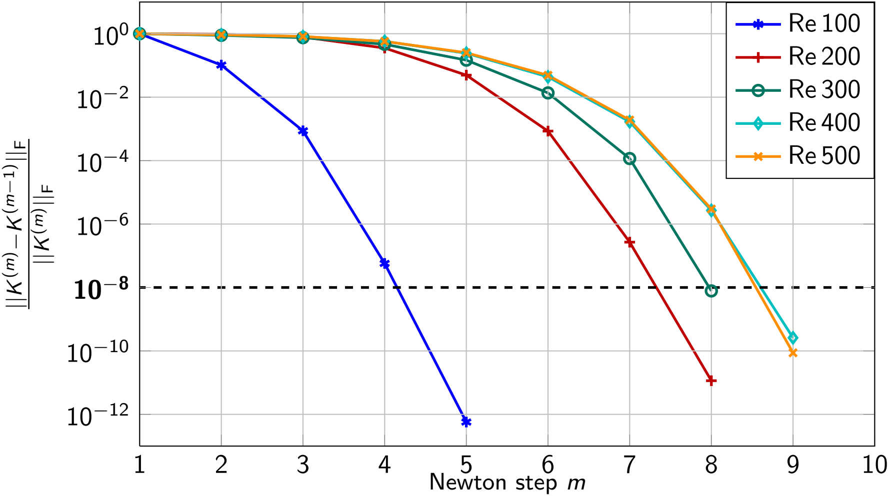
Appendix

Newton-ADI: NSE on von Kármán Vortex Street



Relative change of feedback matrix K for different Reynolds numbers.