

## Workshop on Nonlinear MOR, Schloss Ringberg Tegernsee, 6<sup>th</sup> - 9<sup>th</sup> May 2012



### Sunday, May 6<sup>th</sup>, 2012

05.00 pm Welcome at the conference site

### Monday, May 7<sup>th</sup>, 2012

|                  |                                           |                  |              |
|------------------|-------------------------------------------|------------------|--------------|
| 09.00 – 09.35 am | Stefan Volkwein, University of Konstanz   | 08.00 – 09.00 am | Breakfast    |
| 09.35 – 10.05 am | Tobias Breiten, MPI Magdeburg             |                  |              |
|                  |                                           | 10.05 – 10.35 am | Coffee break |
| 10.35 – 11.10 am | Karen Veroy-Grepl, RWTH Aachen University |                  |              |
| 11.10 – 11.45 am | Gianluigi Rozza, EPF Lausanne             |                  |              |
| 11.45 – 12.15 pm | Martin Heß, MPI Magdeburg                 |                  |              |
|                  |                                           | 12.30 – 02.00 pm | Lunch        |
| 02.00 – 02.35 pm | Heike Faßbender, TU Braunschweig          |                  |              |
| 02.35 – 03.10 pm | Danny Sorensen, Rice University Houston   |                  |              |
|                  |                                           | 03.10 – 03.40 pm | Coffee break |
| 03.40 – 04.15 pm | David Ansallem, Stanford University       |                  |              |
| 04.15 – 04.50 pm | Serkan Gugercin, Virginia Tech Blacksburg |                  |              |
| 04.50 – 05.20 pm | Kapil Ahuja, MPI Magdeburg                |                  |              |
|                  |                                           | 06.30 – 08.00 pm | Dinner       |
| 08.00 – 09.30 pm | Poster session                            |                  |              |

### Tuesday, May 8<sup>th</sup>, 2012

|                  |                                            |                  |              |
|------------------|--------------------------------------------|------------------|--------------|
| 09.00 – 09.35 am | Volker Mehrmann, TU Berlin                 | 08.00 – 09.00 am | Breakfast    |
| 09.35 – 10.10 am | Tobias Damm, University of Bayreuth        |                  |              |
| 10.10 – 10.45 am | Tatjana Stykel, University of Augsburg     |                  |              |
|                  |                                            | 10.45 – 11.15 am | Coffee break |
| 11.15 – 11.50 am | Bernhard Haasdonk, University of Stuttgart |                  |              |
| 11.50 – 12.20 pm | Lihong Feng, MPI Magdeburg                 |                  |              |
|                  |                                            | 12.30 – 02.00 pm | Lunch        |
| 02.00 pm         | Hike                                       |                  |              |
|                  |                                            | 06.30 – 08.00 pm | Dinner       |

### Wednesday, May 9<sup>th</sup>, 2012

|                  |                                      |                  |              |
|------------------|--------------------------------------|------------------|--------------|
| 09.00 – 09.35 am | Michael Hinze, University of Hamburg | 08.00 – 09.00 am | Breakfast    |
| 09.35 – 10.10 am | Michael Mangold, MPI Magdeburg       |                  |              |
| 10.10 – 10.40 am | Sara Grundel, MPI Magdeburg          |                  |              |
|                  |                                      | 10.40 – 11.10 am | Coffee break |
| 11.10 – 12.30 pm | Conclusion                           |                  |              |
|                  |                                      | 12.30 – 02.00 pm | Lunch        |
| 02.00 pm         | Departure                            |                  |              |



### Adaptive POD basis computation using optimal snapshot location

**Stefan Volkwein, University of Koblenz**

The construction of reduced-order models for parameterized partial differential systems using proper orthogonal decomposition (POD) is based on the information of the so-called snapshots. These provide the spatial distribution of the dynamical system at discrete time instances. In this work a strategy is proposed where the POD reduced-order model is improved by choosing additional snapshots in an optimal way. These optimal snapshot locations influence the POD basis functions and therefore the POD reduced-order model. The adaptive POD basis computation is illustrated by the construction of the POD reduced-order model for the complex-valued Helmholtz equation which arises in acoustic applications.

### Two-sided moment matching methods for nonlinear model order reduction

**Tobias Breiten, Max Planck Institute Magdeburg**

In this talk, we will discuss a recently introduced approach for reducing a specific class of nonlinear control systems. The basic idea is to first transform the original model into a so-called quadratic-bilinear control system. Then, by means of variational analysis, these systems will be characterized via a series of generalized transfer functions. We will show how basic tools from tensor theory may be used to improve existing model reduction techniques. This will be done by the construction of two-sided Krylov subspace methods that theoretically lead to better approximations of the generalized transfer functions. Moreover, the tensor framework will allow for an efficient computation of the corresponding projection matrices. New and existent approaches will be compared based on some numerical examples.

### The Reduced Basis Method for Nonlinear Partial Differential Equations

**Karen Veroy-Grepl, RWTH Aachen University**

We present reduced basis (RB) approximation and associated a posteriori error estimation for rapid and reliable solution of parameterized partial differential equations (PDEs). The reduced basis method exploits the parametric structure of the problem and offline-online computational decompositions to obtain real-time response. In this talk we focus in particular on some results and current challenges on the RB method for nonlinear PDEs.

We begin with a review of the main aspects of the methodology for the incompressible stationary Navier-Stokes equations; here we invoke the Brezzi-Rappaz-Raviart theory for analysis of variational approximations of nonlinear partial differential equations to construct rigorous, quantitative, sharp, inexpensive a posteriori error estimators. We also briefly review extensions of this theory to, for example, other types of problems such as nonlinear diffusion. In the second part of this talk we summarize some issues regarding RB for nonlinear PDEs which we currently seek to address. In particular, we discuss issues and challenges in the application of the RB methodology to the Navier-Stokes equations with parameter-dependent geometries, and to problems in nonlinear elasticity.

### Reduced basis methods for parameterized fluid dynamics equations: application to inverse problems in haemodynamics

**Gianluigi Rozza, EPF Lausanne**

Despite the computer resources nowadays available, it is still difficult -- and often impossible -- to deal with applications and scenarios involving the repeated solution of PDEs on different data settings (many-query context) or requiring a numerical solution within a real time context -- or at least very rapidly. For instance, the most common numerical strategies used to tackle optimal control, shape optimization and more general inverse problems under PDE constraints are based on iterative optimization procedures, involving several input/output evaluations as well as many repeated PDE solutions. With this respect, reduced order modeling can represent a suitable strategy to allow for the solution of these problems, entailing an acceptable amount of CPU time and limited storage capacity.

The developed framework deals with inverse problems in fluid dynamics -- such as flow control, shape optimization, inverse identification problems -- and it is based on the coupling between suitable (control, shape, etc.) parameterization strategies and the reduced basis method for the rapid and reliable solutions of parameterized PDEs. We review the current state of the art of the reduced basis method for fluid dynamics equations, with a special emphasis on a posteriori error estimation and Offline/Online decomposition in the Stokes and Navier-Stokes equations in affinely and nonaffinely parameterized geometries, obtained e.g. by means of volume-based shape parameterizations such as free-form deformation or radial basis function techniques. Numerical examples dealing with inverse problems arising in haemodynamics are presented in order to show the capabilities of our reduced framework.

### Reduced Basis Modeling for Parametrized Systems of Maxwell's Equations

**Martin Heß, Max Planck Institute Magdeburg**

The Reduced Basis Method generates low-order models to parametrized PDEs to allow for efficient evaluation of parametrized models in many-query and real-time contexts. The Reduced Basis approach is decomposed into a time-consuming offline phase, which generates a surrogate model and an online phase, which allows fast parameter evaluations. Rigorous and sharp a posteriori error estimators play a crucial role in this process, in that they define which snapshots are to be taken into the reduced space and give bounds to the output quantities during the online phase.

We apply the Reduced Basis Method to systems of Maxwell's equations arising from electrical circuits. Using microstrip models as a microscopic view of interconnect structures, the input-output behaviour is approximated with low order reduced basis models for a parametrized geometry. Parameters under consideration are the distance between microstrips, material coefficients, like

permittivity and permeability of substrates and frequency. The computational obstacle lies in the construction of lower bounds to the inf-sup stability constant, which can be achieved by a successive constraint method.

### Model order reduction for steady aerodynamic applications

#### **Heike Faßbender, TU Braunschweig**

In this talk the fast simulation of air flowing past an airfoil, that is a two-dimensional representation of an airplane wing, is addressed using model order reduction. In particular, reduced order models are used to predict steady aerodynamic flows. At first the governing equations for simulating flows given by the Euler as well as the Navier-Stokes equations are briefly reviewed. Then the key ingredients for the proposed reduced order model approach, the Proper Orthogonal Decomposition [1] and the Missing Point Estimation [2], are explained as well as the method proposed here. Finally, numerical results for a high-lift wing-flap configuration are presented. Such configurations are used in the take-off and landing phase of flight and are therefore highly relevant for industry.

[1] P. Holmes, J.L. Lumley and G. Berkooz: Turbulence, Coherent Structures, Dynamical Systems and Symmetry, Cambridge, New York 1996.

[2] P. Astrid, S. Weiland, K. Willcox and T. Backx: Missing point estimation in models described by proper orthogonal decomposition, IEEE Transactions on Automatic Control, Vol. 53, No. 10, pp. 2237-2251, 2008.

### MOR for Bratu Problems using DEIM

#### **Danny C. Sorensen, Rice University Houston**

The Discrete Empirical Interpolation Method (DEIM) is a method for nonlinear model order reduction that is a modification of proper orthogonal decomposition (POD) that addresses a serious complexity issue. This POD-DEIM approach has provided spectacular dimension and complexity reduction for challenging systems large scale ordinary differential equations (ODEs). Reductions from 15,000 variables to 40 variables in the reduced model with very little loss of accuracy have been achieved. The DEIM is surprisingly simple and amounts to replacing orthogonal projection with an interpolatory projection of the nonlinear term that only requires the evaluation of a few selected components of the nonlinear term.

We have applied this approach to dimension reduction of Bratu type problems. For the standard Bratu problem, the results are very encouraging and have the surprising property of capturing the critical turning point without taking snapshots in the turning point region. The addition of a total variation term has led to a more challenging problem and the implications of this for general problems will be discussed.

### Nonlinear model reduction based on local reduced-order bases

#### **David Amsallem, Stanford University**

A new approach for the dimensional reduction via projection of nonlinear computational models based on the concept of local reduced-order bases is presented. It is particularly suited for problems characterized by different physical regimes, parameter variations, or moving features such as discontinuities and fronts. Instead of approximating the solution of interest in a fixed lower-dimensional subspace of global basis vectors, the proposed model order reduction method approximates this solution in a lower-dimensional subspace generated by most appropriate local basis vectors. To this effect, the solution space is partitioned into sub-regions and a local reduced-order basis is constructed and assigned to each sub-region offline. During the incremental solution online of the reduced problem, a local basis is chosen according to the sub-region of the solution space where the current high-dimensional solution lies. This is achievable in real-time because the computational complexity of the selection algorithm scales with the dimension of the lower-dimensional solution space. Because it is also applicable to the process of hyper reduction, the proposed method for nonlinear model order reduction is computationally efficient. Its potential for achieving large speedups while maintaining good accuracy is demonstrated for two nonlinear computational fluid and fluid-structure interaction problems.

### Interpolation and H2-model reduction of Bilinear Systems

#### **Serkan Gugercin, Virginia Tech Blacksburg**

Authors: Garret Flagg and Serkan Gugercin

Bilinear systems are an important class of weakly nonlinear systems and appear in several applications. The interpolatory model reduction methods for bilinear systems so far have focused on interpolating the homogenous subsystems individually. In this talk, we introduce a new framework where multi-point interpolation is performed on the underlying full Volterra series. This, in turn, has a direct connection to optimal-H2 model reduction of bilinear systems and allows us to produce optimal-H2 approximants by solving only regular Sylvester equations.

### Krylov Subspace Recycling for Interpolatory Model Order Reduction

#### **Kapil Ahuja, Max Planck Institute Magdeburg**

Interpolatory model order reduction algorithms require solving sequences of large sparse linear systems. Krylov subspace recycling is a technique for efficiently solving such systems. We present recycling methods for both non-parametric and parametric model order reduction. The recycle spaces are approximate left and right invariant subspaces corresponding to the eigenvalues close to the origin. For the non-parametric case, we show that solving a problem without recycling leads to about 50% increase in run time. In-general, we show savings of up to 70% in the iteration count.

### Direct discretization of input/output maps for flow control problems

**Volker Mehrmann** (joint with Jan Heiland), **TU Berlin**

A classical approach for control problems associated with fluid dynamics problems is to first discretize the forward problem and then carry out a model reduction to reduce the system size to a level where optimization and feedback control problems can be applied. As an alternative we present a discretization method for the input/output map which allows to generate a small size state-space transfer function representation. This allows to control the error in the space-time approximation together with the error in the input and output approximation.

### Balanced Truncation of Linear Stochastic Systems

**Tobias Damm**, **University of Bayreuth**

We consider linear stochastic differential systems and define a new controllability Gramian. Using this Gramian, we apply balanced truncation in the usual way. Then we show that the approximation error in the input-output norm can be bounded by twice the sum of the neglected generalized Hankel singular values. Some aspects of the approach are still open and will be discussed.

### Model reduction based optimal control for field-flow fractionation

**Tatjana Stykel**, **Carina Willbold**, **University of Augsburg**

We discuss the application of model order reduction to optimal control problems governed by coupled systems of the Stokes-Brinkman and advection-diffusion equations. Such problems arise in field-flow fractionation processes for the efficient and fast separation of particles of different size in microfluidic flows.

Our model reduction approach is based on a combination of balanced truncation and tangential interpolation for model reduction of the semidiscretized optimality system. Numerical results demonstrate the properties of this approach.

### Kernel-Based MOR for Nonlinear Parameterized Systems

**Bernard Haasdonk**, **University of Stuttgart**

We consider model order reduction for parameterized nonlinear systems. The dynamical systems under consideration consist of a nonlinear, time- and parameter-dependent kernel expansion representing the system's inner dynamics as well as time- and parameter-affine inputs, initial conditions and outputs. The reduction technique we use was originally proposed by (Phillips et al. 2003) which we extend here to the full parametric and time-dependent case. The system is reduced applying a Galerkin projection with biorthogonal matrices obtained by the POD-Greedy procedure (Haasdonk 2011). By using translation- and rotation-invariant kernels, we can avoid the reconstruction step required in the evaluation of the nonlinearity in standard projection-based reduction techniques. We also can provide rigorous and efficient a-posteriori error estimation for the reduction error in state-space and output (Wirtz & Haasdonk 2012). Key features for this are to use local Lipschitz constant estimates and an iterative scheme to balance computation costs against estimation sharpness. A full offline/online decomposition for both the reduction process and the error estimators can be provided in case of time- and parameter-affine system components. This results in both rapid reduced simulation and error estimation. Experimental results demonstrate the applicability of the reduction technique.

### Model reduction and ROM-based optimization of simulated moving bed chromatography

**Lihong Feng**, **Max Planck Institute Magdeburg**

Simulated moving bed (SMB) chromatography is an effective adsorption process. Due to its intrinsic periodic behavior and complex nonlinear process dynamics, optimization based on rigorous first-principles SMB models is typically a computationally expensive task. In this talk, we explore the use of cheaper reduced order models (ROMs) to address associated computational challenges present in the steady state optimization of SMB processes. In this new optimization approach, ROMs are derived using proper orthogonal decomposition (POD) technique. The trust region framework is applied to manage the constructed POD surrogates and to guarantee convergence to the optimum of the original full order model. A separation of two enantiomers characterized by nonlinear bi-Langmuir isotherms is considered to evaluate the effectiveness of the proposed scheme. The ROM-based solution method is compared to that of the conventional full order optimization in terms of solution accuracy, computing time, etc. The significant advantages gained by using ROMs for optimization is illustrated.

### Parametric model order reduction of electrical networks

**Michael Hinze**, joint with Martin Kunkel, Ulrich Matthes, Andreas Steinbrecher, and Tatjana Stykel  
**University of Hamburg**

We consider electrical networks with semiconductors modeled by modified nodal analysis and drift-diffusion (DD) equations. Discretization of the DD-equations yields a high dimensional DAE system. We show how proper orthogonal decomposition (POD) for the semiconductors combined with PABTEC for the network can be used to reduce the dimension of the model. Furthermore we discuss a greedy approach to construct overall models which are valid over certain parameter ranges.

### Reduced Models for the Control of Particle Processes in Non-Ideal Fluid Flows

**Michael Mangold**, **Max Planck Institute Magdeburg**

Particle processes are of great importance in chemical and pharmaceutical industry.

Crystallization, granulation, or polymerization are prominent examples. Often, the evolution of particle populations is strongly affected by the flow conditions in the fluid phase (aggregation, breakage, attrition etc.). Process models that account for the relevant physical effects in an adequate way comprise Navier Stokes equations, energy balances, and mass balances for the liquid phase, as well as population balance equations for the particle phase. They are distributed in several external (space) and internal (property) coordinates and are computationally very demanding. A direct application of such models to process control and process design problems is hardly feasible due to the high computational burden. Hence there is a need for reduced control oriented models of low system order.

A widely used approach in control application is to approximate the fluid phase by idealizing physical assumptions like perfect mixing or compartment models. The resulting inaccuracy of the process models is compensated by conservative / robust controller design. A reduced model of the particle phase is usually obtained by applying the method of moments. However, it is well-known that one cannot reconstruct the complete particle distribution from a finite number of moments and that the method of moments is unsatisfactory for multi-modal distributions. More systematic reduction techniques are desirable.

In this contribution, model reduction by Proper Orthogonal Decomposition is suggested for population balance systems coupled with fluid dynamics. While POD is a well-established approach for model reduction of Navier Stokes equations, it has hardly been applied so far to particle systems. One of the additional difficulties compared to pure Navier Stokes equations are the more complicated nonlinear terms. An efficient and accurate treatment of the nonlinearities during runtime of the reduced model is non-trivial. It is found that the best point interpolation method recently introduced by Nguyen, Patera et al. for parameterized functions is an elegant way to solve this problem.

### *Multivariate Approximation in the context of Model Order Reduction*

**Sara Grundel, Max Planck Institute Magdeburg**

We will look at some standard Model Order Reduction techniques and interpret them as multivariate approximations, where the transfer function or the state in the frequency domain is given by a linear combination of ansatz functions in the frequency  $s$ . This is just an abstract interpretation but by modifying this ansatz one can generalize it to nonlinear and parametric problems. In this context we will explore Radial Basis Functions and Kriging. These are two related successful techniques in multivariate approximation of scattered data, where the approximate is a linear combination of given ansatz functions that are typically centered around the data points, often called snapshots.

## Participants

### *External groups*

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