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Efficient Methods for Reduced Order State Space Modeling of Piezo-Mechanical Systems

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MAX PLANCK INSTITUTE
FOR DYNAMICS OF COMPLEX
TECHNICAL SYSTEMS
MAGDEBURG

Motivation

Adaptive Spindel Support (ASS) with Piezo Actuators

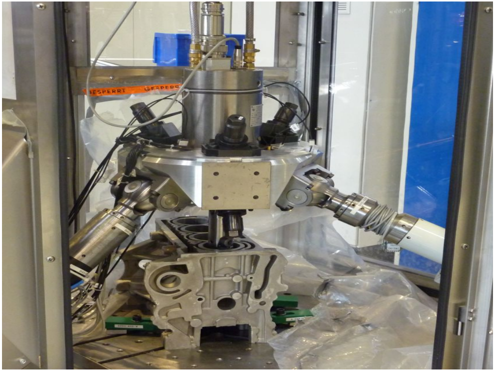

Figure: (a) ASS mounted in (b) Parallel-Kinematic Machine (PKM)

Source: B. Kranz, Fraunhofer IWU, Dresden, Germany.

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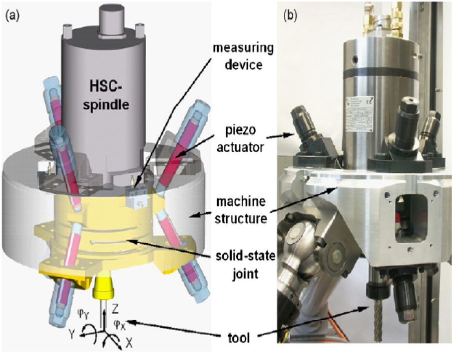

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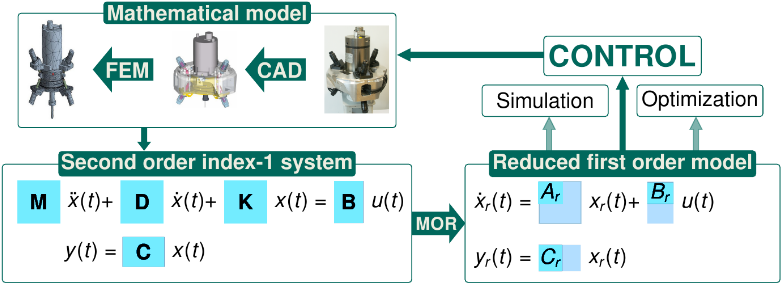
Simulation, Design Optimization and Controller Design



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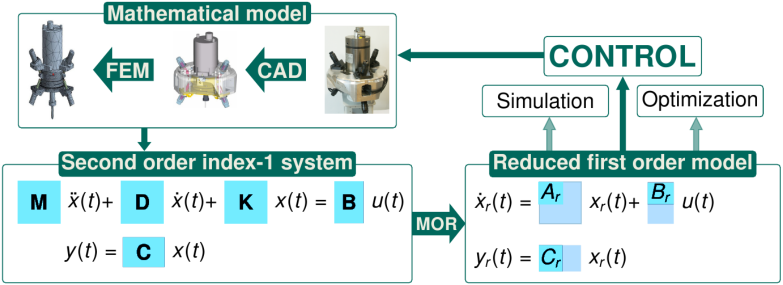

Primary goal: state space ROM for use in, e.g., Simulink

Outline

Balanced Truncation for Large Scale Systems

BT Basics

- Given LTI continuous-time system

$$\Sigma : \boxed{\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t) + Du(t),}$$

where $x(t) \in \mathbb{R}^n, u(t) \in \mathbb{R}^m, y(t) \in \mathbb{R}^p$ and A, B, C and D are matrices.
- The realization (A, B, C), of the system Σ, is called balanced, if the solutions P, Q of the Lyapunov equations

$$\boxed{AP + PA^T + BB^T \; = \; 0, \qquad A^T Q + QA + C^T C \; = \; 0,}$$

satisfy: $P = Q = \mathrm{diag}(\sigma_1, \ldots, \sigma_n)$ where $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_n > 0$.
- $\{\sigma_1, \ldots, \sigma_n\}$ are the Hankel singular values (HSVs) of Σ.
- A balanced realization is computed via state space transformation

$$\begin{aligned} \mathcal{T} : (A, B, C) &\mapsto (TAT^{-1}, TB, CT^{-1}) \\ &= \left(\begin{bmatrix} A_k & A_2 \\ A_3 & A_4 \end{bmatrix}, \begin{bmatrix} B_k \\ B_2 \end{bmatrix}, \begin{bmatrix} C_k & C_2 \end{bmatrix} \right). \end{aligned}$$

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- Truncation $\rightsquigarrow k(\ll n)$ dimensional reduced order model:

$$\hat{\Sigma} : \boxed{\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t), \quad \hat{y}(t) = \hat{C}\hat{x}(t) + Du(t),}$$

where $(\hat{A}, \hat{B}, \hat{C}) = (A_k, B_k, C_k)$.

- Such that $\|y - \hat{y}\|_\infty$ or $\|G - \hat{G}\|_\infty$ is small enough.

Balanced Truncation for Large Scale Systems

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- Truncation $\rightsquigarrow k(\ll n)$ dimensional reduced order model:

Transfer function:

$G(s) = C(sI - A)^{-1}B + D$, where $s \in \mathbb{C}$, I is identity matrix.

where $(\hat{A}, \hat{B}, \hat{C}) = (A_k, B_k, C_k)$.

- Such that $\|y - \hat{y}\|_\infty$ or $\|G - \hat{G}\|_\infty$ is small enough.

Balanced Truncation for Large Scale Systems

Implementation

The SR Method

1. Compute (Cholesky) factors of the solutions to the Lyapunov equation,
$$P = S^T S, \quad Q = R^T R.$$
2. Compute singular value decomposition
$$SR^T = [\, U_1,\ U_2 \,] \begin{bmatrix} \Sigma_1 & \\ & \Sigma_2 \end{bmatrix} \begin{bmatrix} V_1^T \\ V_2^T \end{bmatrix}.$$
3. Define
$$W := R^T V_1 \Sigma_1^{-1/2} \in \mathbb{R}^{n\times k} \qquad V := S^T U_1 \Sigma_1^{-1/2} \in \mathbb{R}^{n\times k}.$$
4. Then the reduced order model $\hat{\Sigma} = (\hat{A}, \hat{B}, \hat{C}) = (W^T A V, W^T B, C V)$.

Solving Large Lyapunov Equations

LRCF-ADI

[BENNER/LI/PENZL '08]

Given $FX + XF^T = -GG^T \qquad F \in \mathbb{R}^{n\times n}, G \in \mathbb{R}^{n\times p}$

Task Find $Z \in \mathbb{C}^{n,nz}$, such that $nz \ll n$ and $X \approx ZZ^H$

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Task Find $Z \in \mathbb{C}^{n,nz}$, such that $nz \ll n$ and $X \approx ZZ^H$

Algorithm

$$V_1 = \sqrt{-2\,\mathrm{Re}\,(p_1)}(F + p_1 I)^{-1}G, \qquad Z_1 = V_1$$

$$V_i = \frac{\sqrt{\mathrm{Re}\,(p_i)}}{\sqrt{\mathrm{Re}\,(p_{i-1})}}\left[I - (p_i + \overline{p_{i-1}})(F + p_i I)^{-1}\right] V_{i-1}, \qquad Z_i = [Z_{i-1} V_i]$$

- For certain shift parameters $\{p_1, ..., p_J\} \subset \mathbb{C}^-$.
- Stop the algorithm if $\|FZ_iZ_i^H + Z_iZ_i^H F^T + GG^T\|$ is small.

Solving Large Lyapunov Equations

G-LRCF-ADI (E invertible) [SAAK '09]

Given $FXE^T + EXF^T = -GG^T \qquad E, F \in \mathbb{R}^{n\times n}, G \in \mathbb{R}^{n\times p}$

Task Find $Z \in \mathbb{C}^{n,nz}$, such that $nz \ll n$ and $X \approx ZZ^H$

Algorithm

$$V_1 = \sqrt{-2\,\mathrm{Re}\,(p_1)}(F + p_1 E)^{-1} G, \qquad Z_1 = V_1$$

$$V_i = \frac{\sqrt{\mathrm{Re}\,(p_i)}}{\sqrt{\mathrm{Re}\,(p_{i-1})}} \left[I - (p_i + \overline{p_{i-1}})(F + p_i E)^{-1}\right] E V_{i-1}, \quad Z_i = [Z_{i-1}\, V_i]$$

- For certain shift parameters $\{p_1, ..., p_J\} \subset \mathbb{C}^-$.
- Stop the algorithm if $\|FZ_iZ_i^HE^T + EZ_iZ_i^HF^T + GG^T\|$ is small.

Solving Large Lyapunov Equations

S-LRCF-ADI (index 1) [Rommes/Freitas/Martins '08]

Index 1 system

$$\begin{bmatrix} E_{11} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u(t)$$

Given $\tilde{F} X E_{11}^T + E_{11} X \tilde{F}^T = -\tilde{G}\tilde{G}^T, \qquad E_{11}, \tilde{F} \in \mathbb{R}^{n \times n}, \tilde{G} \in \mathbb{R}^{n \times p}$

$$\tilde{F} = F_{11} - F_{12} F_{22}{}^{-1} F_{21}, \quad \tilde{G} = B_1 - F_{12} F_{22}{}^{-1} B_2$$

Task: Find $Z \in \mathbb{C}^{n,nz}$, such that $nz \ll n$ and $X \approx ZZ^H$

Algorithm

$$\begin{bmatrix} V_1 \\ * \end{bmatrix} = \sqrt{-2\,\mathrm{Re}\,(p_1)} \begin{bmatrix} F_{11} + p_1 E_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \qquad Z_1 = V_1$$

$$\begin{bmatrix} V_i \\ * \end{bmatrix} = \frac{\sqrt{\mathrm{Re}\,(p_i)}}{\sqrt{\mathrm{Re}\,(p_{i-1})}} \left[I - (p_i + \overline{p_{i-1}}) \begin{bmatrix} F_{11} + p_i E_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}^{-1} \right] \begin{bmatrix} E_{11} V_{i-1} \\ 0 \end{bmatrix}, \qquad Z_i = [Z_{i-1} V_i]$$

Solving Large Lyapunov Equations

S-LRCF-ADI **(Real low-rank factor)** [Benner/Kürschner/Saak '12]

Given $\tilde{F}XE_{11}^T + E_{11}X\tilde{F} = -\tilde{G}\tilde{G}^T, \qquad E_{11}, \tilde{F} \in \mathbb{R}^{n\times n}, \tilde{G} \in \mathbb{R}^{n\times p}$

$\tilde{F} = F_{11} - F_{12}F_{22}^{-1}F_{21}, \quad \tilde{G} = B_1 - F_{12}F_{22}^{-1}B_2$

Task Find $Z \in \mathbb{R}^{n,nz}$, such that that $nz \ll n$ and $X \approx ZZ^H$

Algorithm

$$Z_0 = []$$

$$\begin{bmatrix} V_1 \\ * \end{bmatrix} = \sqrt{-2\,\mathrm{Re}\,(p_1)} \begin{bmatrix} F_{11} + p_1 E_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}^{-1} \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$

$$\begin{bmatrix} V_i \\ * \end{bmatrix} = \frac{\sqrt{\mathrm{Re}\,(p_i)}}{\sqrt{\mathrm{Re}\,(p_{i-1})}} \left[I - (p_i + \overline{p_{i-1}}) \begin{bmatrix} F_{11} + p_i E_{11} & F_{12} \\ F_{21} & F_{22} \end{bmatrix}^{-1} \right] \begin{bmatrix} E_{11} V_{i-1} \\ 0 \end{bmatrix}$$

IF $\mathrm{Im}(p_i) = 0, \; Z_i = [Z_{i-1}, V_i]$, ELSE $\beta = 2\frac{\mathrm{Re}(p_i)}{\mathrm{Im}(p_i)}, \; V_{i+1} = \overline{V}_i + \beta \mathrm{Im}(V_i)$

$$Z_{i+1} = [Z_{i-1}, \sqrt{2}\mathrm{Re}(V_i) + \frac{\beta}{\sqrt{2}}\mathrm{Im}(V_i), \sqrt{\frac{\beta^2}{2} + 2}.\mathrm{Im}(V_i)]$$

Solving Large Lyapunov Equations

Efficient Solvers for Second Order Systems [Benner/Kürschner/Saak '12]

Second Order Form

$$M\ddot{x} + D\dot{x} + Kx = Bu$$

First Order Form

$$\mathcal{E}\dot{z} = \mathcal{A}z + \mathcal{B}u$$

- x displacements,
- $z = (\dot{x}^T, x^T)^T$,
- M,D,K invertible,
- F arbitrary but invertible.

$$\mathcal{E} = \begin{bmatrix} 0 & F \\ M & D \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} F & 0 \\ 0 & -K \end{bmatrix},$$

$$\mathcal{B} = \begin{bmatrix} 0 \\ B \end{bmatrix}.$$

Solving Large Lyapunov Equations

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G-LRCF-ADI

main task per step: $(\mathcal{A} + p_i\mathcal{E})x = \mathcal{E}f, \quad x = [x_1, x_2]^T$

Solving Large Lyapunov Equations

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main task per step: $(\mathcal{A} + p_i\mathcal{E})x = \mathcal{E}f, \quad x = [x_1, x_2]^T$

SO-LRCF-ADI

$$(p_i^2 M - p_i D + K)x_2 = (p_i M - D)f_2 - Mf_1, \qquad x_1 = f_2 - p_i x_2.$$

MOR for Piezo-Mechanical Systems

Problem Structure

Equation of Motion (from FEM, here ANSYS)

$$M\ddot{\xi}(t) + D\dot{\xi}(t) + K\xi(t) = Qu(t),$$

where $M, D, K \in \mathbb{R}^{n_1 \times n_1}$ and $Q \in \mathbb{R}^{n_1 \times p}$.

Properties

- M, D, K are mass, damping and stiffness matrices, respectively.
- All matrices sparse.
- M and D are singular.
- $p \ll n_1$.

MOR for Piezo-Mechanical Systems

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Equation of Motion (from FEM, here ANSYS)

$$M\ddot{\xi}(t) + D\dot{\xi}(t) + K\xi(t) = Qu(t).$$

Reordering

$$\begin{bmatrix} M_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{z}(t) \\ \ddot{\varphi}(t) \end{bmatrix} + \begin{bmatrix} D_1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{z}(t) \\ \dot{\varphi}(t) \end{bmatrix} + \begin{bmatrix} K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix} \begin{bmatrix} z(t) \\ \varphi(t) \end{bmatrix} = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} u(t).$$

- Second-order index-1 system since K_{22} is invertible.
- $z(t)$ and $\varphi(t)$ are vectors of mechanical displacements and electrical potentials.
- $M_1, D_1, K_{11} \in \mathbb{R}^{n_d \times n_d}$, and $K_{22} \in \mathbb{R}^{n_a \times n_a}$ where $n_d + n_a = n_1$.

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State-Space Form $(x_1 = \dot{z}, x_2 = z)$

$$\underbrace{\begin{bmatrix} 0 & \mathcal{F} & 0 \\ M_1 & D_1 & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\mathcal{E}} \begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \\ \dot{\varphi}(t) \end{bmatrix} = \underbrace{\begin{bmatrix} \mathcal{F} & 0 & 0 \\ 0 & -K_{11} & -K_{12} \\ 0 & -K_{12}^T & -K_{22} \end{bmatrix}}_{\mathcal{A}} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \varphi(t) \end{bmatrix} + \underbrace{\begin{bmatrix} 0 \\ Q_1 \\ Q_2 \end{bmatrix}}_{\mathcal{B}} u(t),$$

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State-Space Form $(x_1 = \dot{z}, x_2 = z)$

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$$y(t) = \underbrace{\begin{bmatrix} 0 & G_1 & G_2 \end{bmatrix}}_{\mathcal{C}} \begin{bmatrix} x_1(t) \\ x_2(t) \\ \varphi(t) \end{bmatrix}.$$

Advantages

- $Q_1 = G_1^T$, $Q_2 = G_2^T \Rightarrow \mathcal{B} = \mathcal{C}^T$,
- $\mathcal{F} = M_1 \Rightarrow \mathcal{E}, \mathcal{A}$ symmetric.

$\Rightarrow$ System symmetric $\Rightarrow$ Lyapunov equations coincide.

MOR for Piezo-Mechanical Systems

Problem Structure

Linear System Inside the ADI

$$\begin{bmatrix} M_1 & \mu_1 M_1 & 0 \\ \mu_1 M_1 & \mu_1(D_1 - K_{11}) & -K_{12} \\ 0 & -K_{12}^T & -K_{22} \end{bmatrix} \begin{bmatrix} x_1^{(i)} \\ x_2^{(i)} \\ \Lambda \end{bmatrix} = \begin{bmatrix} 0 \\ f_1 \\ f_2 \end{bmatrix}.$$

- Solve $\begin{bmatrix} \mu_1^2 M_1 - \mu_1 D_1 + K_{11} & K_{12} \\ K_{12}^T & K_{22} \end{bmatrix} \begin{bmatrix} x_2^{(i)} \\ \Lambda \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix}$ for $x_2^{(i)}$.
- If $i = 1$, $f_1 = -B_1, f_2 = -B_2$ and $x_1^{(1)} = -\mu_1 x_2^{(1)}$.
- Otherwise, $f_1 = (p_i V_{i-1}^2 - V_{i-1}^1)M_1 + D_1 V_{i-1}^2, f_2 = 0$ and $x_1^{(i)} = -p_i x_2^{(i)} + V_{i-1}^2$. $V_{i-1} = \begin{bmatrix} V_{i-1}^1 \\ V_{i-1}^2 \end{bmatrix}$ (see ADI methods).

Numerical Results

ASS Model (Fraunhofer IWU)

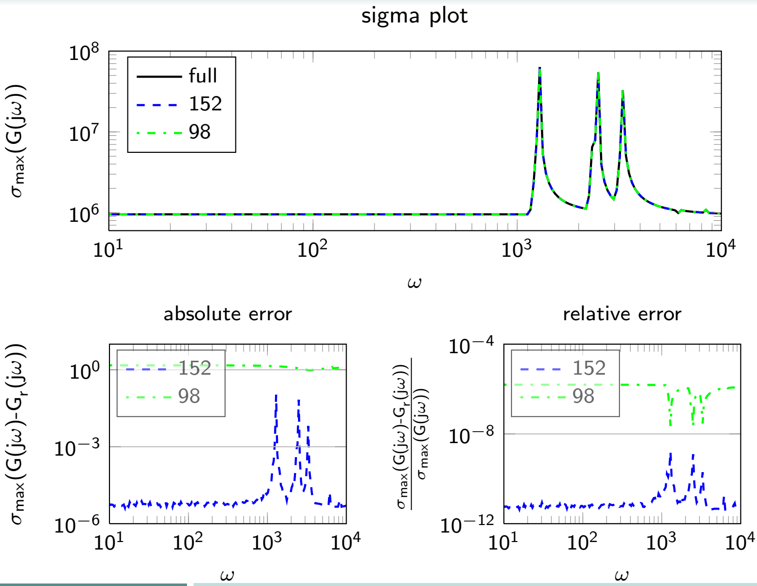
ADI shifts			ADI iterations	system dimension	
k_m	k_p	total	$Z \in R^{n,nz}$	original	reduced
60	50	40	400	290 137	≤ 152

- Number of inputs/outputs: 9
- Dimension of reduced order models versus error bounds:

MOR tolerance	ROM dimension
10^{-5}	152
10^{-4}	146
10^{-3}	140
10^{-2}	132
10^{-1}	123
10	98

Numerical Results

Sigma plot (maximum singular values)



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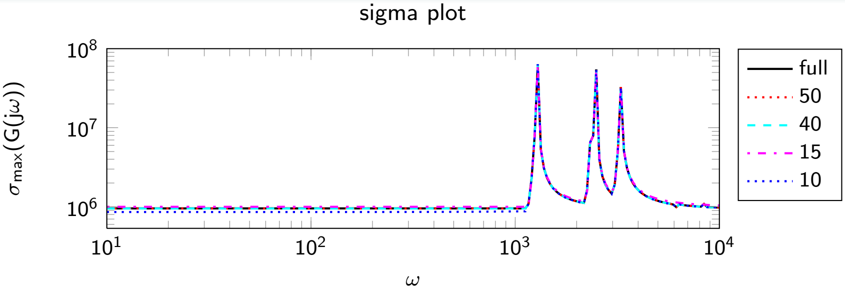
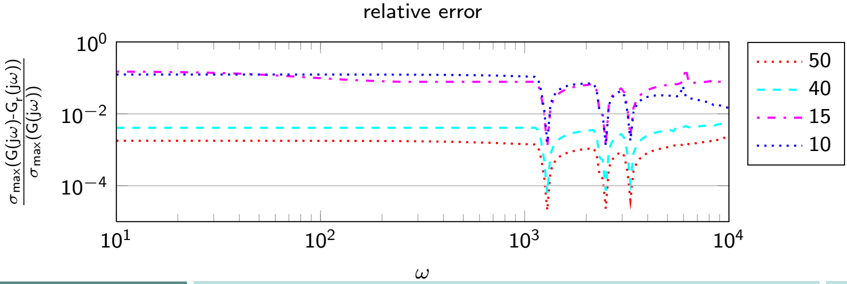
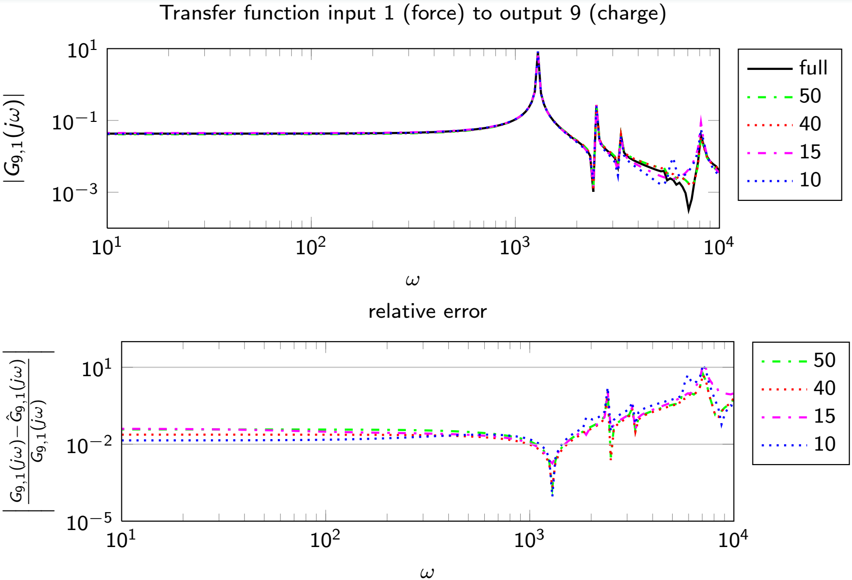




Numerical Results

Single input to single output relation



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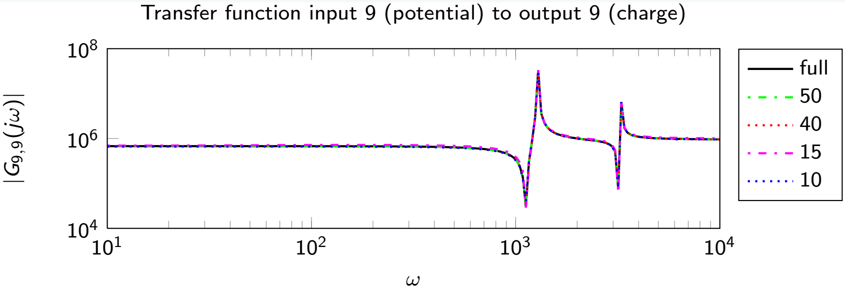

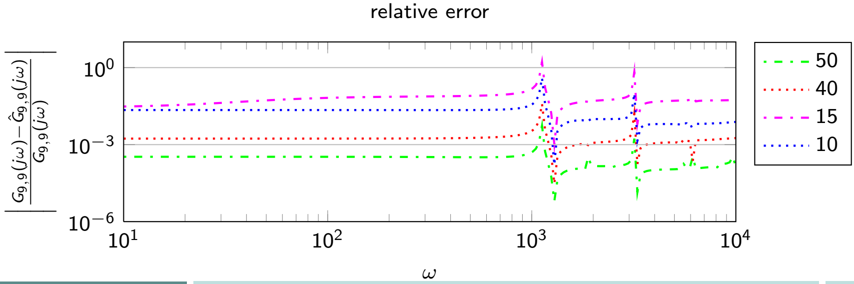

Numerical Results

Comparison reduced models with full model

system dimension	execution time (sec)	speedup
290 137	90.00	1
152	0.0162	5 555
98	0.0049	18 367
50	0.0025	36 000
40	0.0018	50 000
15	0.0013	69 230
10	0.0009	100 000

Table: Average execution time and speedup against full order model for computing the maximum Hankel singular value at a given sampling frequency.

Hardware - software setup

- MATLAB®7.11.0 (R2010b),
- 4 Intel® Xeon® E7-8837 CPUs with a 2.67-GHz clock, 8 Cores each,
- 1TB of total RAM.

Conclusion and Outlook

Overview

- Efficient model reduction method is developed for second order index 1 systems, and applied to ASS model.
- The accuracy of the method is demonstrated by a frequency domain error analysis.
- Even very low order surrogate models (10dof) preserve the main features of the transfer behavior of the full (290137dof) FEM model.
- They are expected to perform well in controller design.

Future work

- Implicit handling of higher indexes, e.g., structural dynamics with holonomic constraints.

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Thank you for your attention!

References

- Benner, P., Kürschner, P., Saak, J.: Efficient Handling of Complex Shift Parameters in the Low-Rank Cholesky Factor ADI Method.
  Numerical Algorithms pp. 1–27 (2012).
- Freitas, F., Rommes, J., Martins, N.: Gramian-based reduction method applied to large sparse power system descriptor models.
  IEEE Transactions on Power Systems **23**(3), 1258–1270 (2008)
- Neugebauer, R., Drossel, W.G., Bucht, A., Kranz, B., Pagel, K.: Control design and experimental validation of an adaptive spindle support for enhanced cutting processes.
  CIRP Annals - Manufacturing Technology **59**(1), 373–376 (2010).
- Saak, J. : Efficent numerical solution of large scale algebraic matrix equations in PDE control and model order reduction.
  PhD thesis, Chemnitz University of Technology, Chemnitz, Germany,2009.
- Uddin, M. M., Saak, J., Kranz, B. and Benner, P.: Computation of a Compact State Space Model for a Spindle Head Configuration with Piezo Actuators using Balanced Truncation.
  Production Engineering Research and Development, to appear (2012).