Model Order Reduction of Affinely Parameter-dependent Systems Using Interpolation on Grassmann Manifolds

MoreSim_4_Nano

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Problem Overview

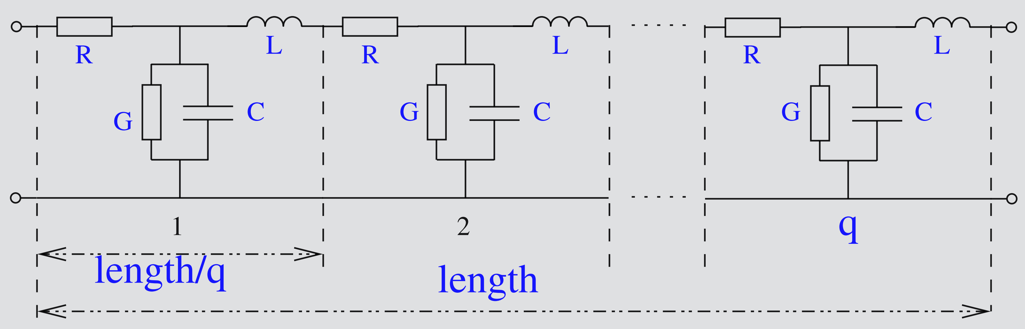
Parametric linear circuit equations:

$$E(p)\dot{x}(t) = A(p)x(t) + Bu(t),$$
$$y(t) = B^T x(t),$$

$$E(p) = \begin{bmatrix} A_C C(p) A_C^T & 0 & 0 \\ 0 & L(p) & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad A(p) = \begin{bmatrix} -A_R G(p) A_R^T & -A_L & -A_V \\ A_L^T & 0 & 0 \\ A_V^T & 0 & 0 \end{bmatrix},$$

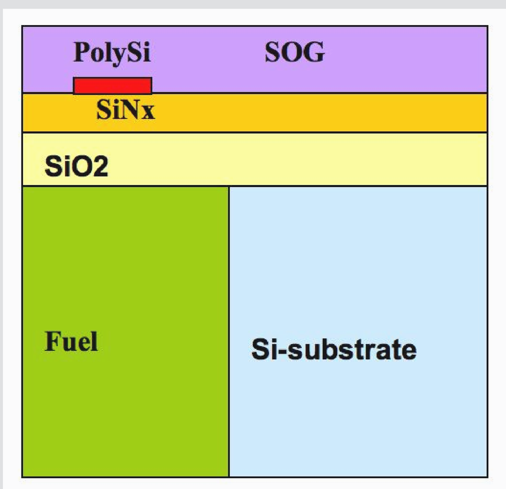
$$B = \begin{bmatrix} -A_I & 0 \\ 0 & 0 \\ 0 & -I \end{bmatrix}, \; x = \begin{bmatrix} \eta \\ i_L \\ i_V \end{bmatrix}, \; u = \begin{bmatrix} i_I \\ u_V \end{bmatrix}, \; y = -\begin{bmatrix} u_I \\ i_V \end{bmatrix}, \; p \in \mathbb{P} \subset \mathbb{R}^d.$$

- A_*, C, L, G: incidence, capacitance, inductance, conductance matrices



Microthruster unit ([Lasance '01, Rudnyi/Korvink '08])

$$E\dot{T}(t) = (A - h_{top}A_{top} - h_{bot}A_{bot} - h_{sid}A_{side})T(t) + Bu(t),$$
$$y(t) = CT(t).$$



Parametric model order redcution (PMOR)

- Many large-scale parameter-dependent systems need to be simulated. Due to the high order of the models, model order reduction is required.
- Standard model order reduction methods and interpolation can be used; the reduced order models for new points should be able to be fast computed without doing model order reduction again.

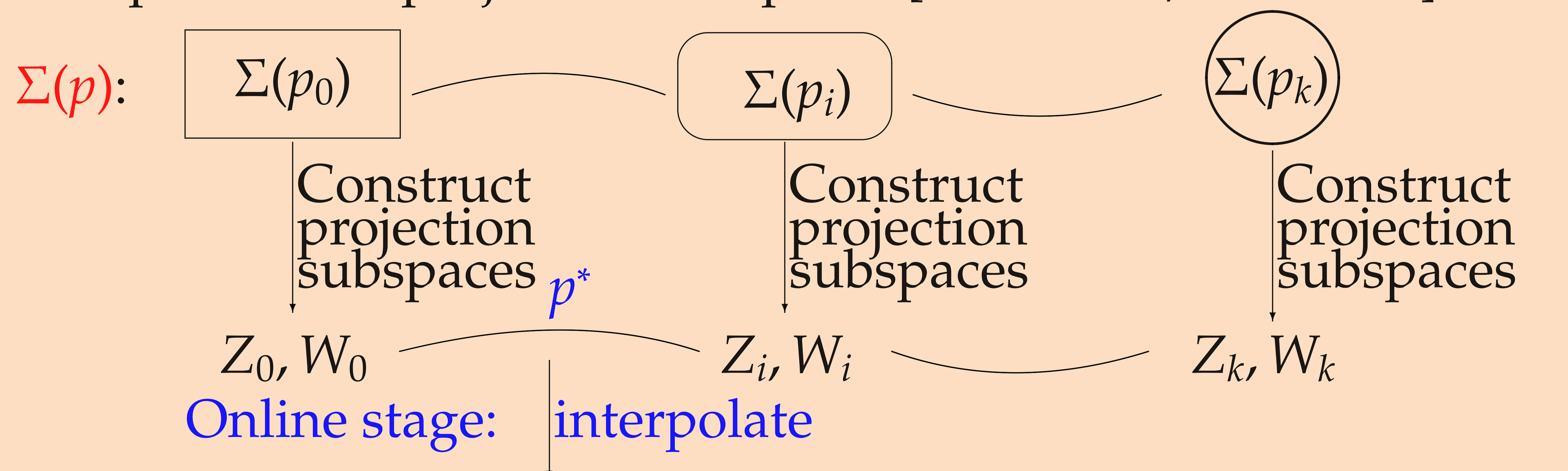
$$E(p)\dot{x}(t) = A(p)x(t) + B(p)u(t)$$
$$y(t) = C(p)x(t)$$

PMOR →

$$\hat{E}(p)\dot{\hat{x}}(t) = \hat{A}(p)\hat{x}(t) + \hat{B}(p)u(t)$$
$$\hat{y}(t) = \hat{C}(p)\hat{x}(t)$$

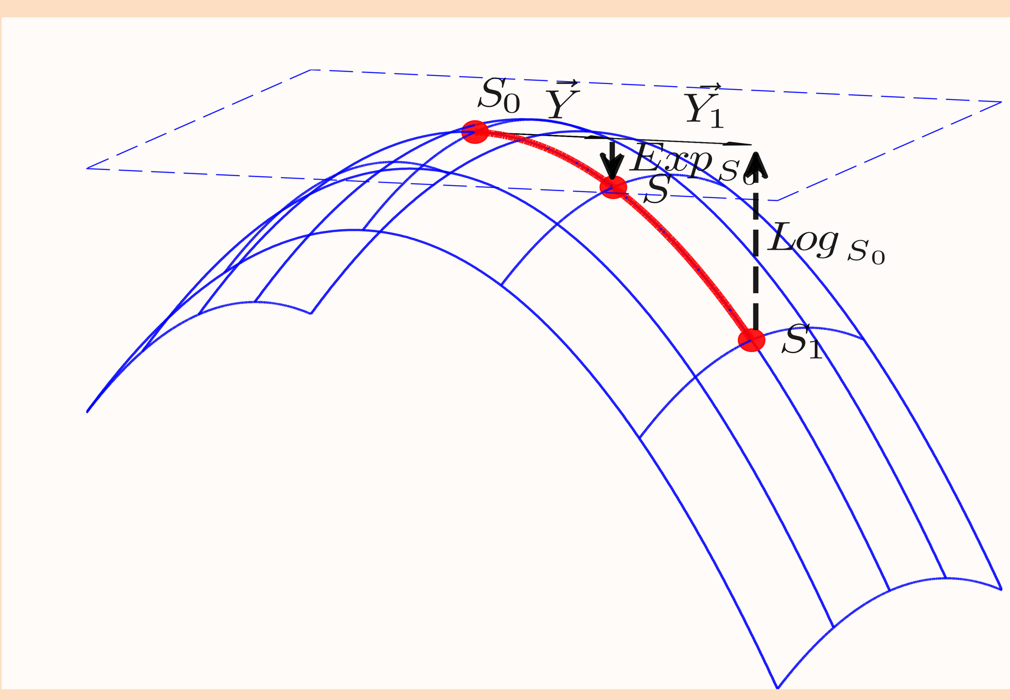
Method

- Interpolation of projection subspaces [Amsallem/Farhat '08]



$$\left(\begin{array}{c|c|c} Z(p^*)^T E(p^*) W(p^*) & Z(p^*)^T A(p^*) W(p^*) & Z(p^*)^T B(p^*) \\ & C(p^*) W(p^*) & \end{array}\right)$$

- Interpolation on Grassmann manifolds



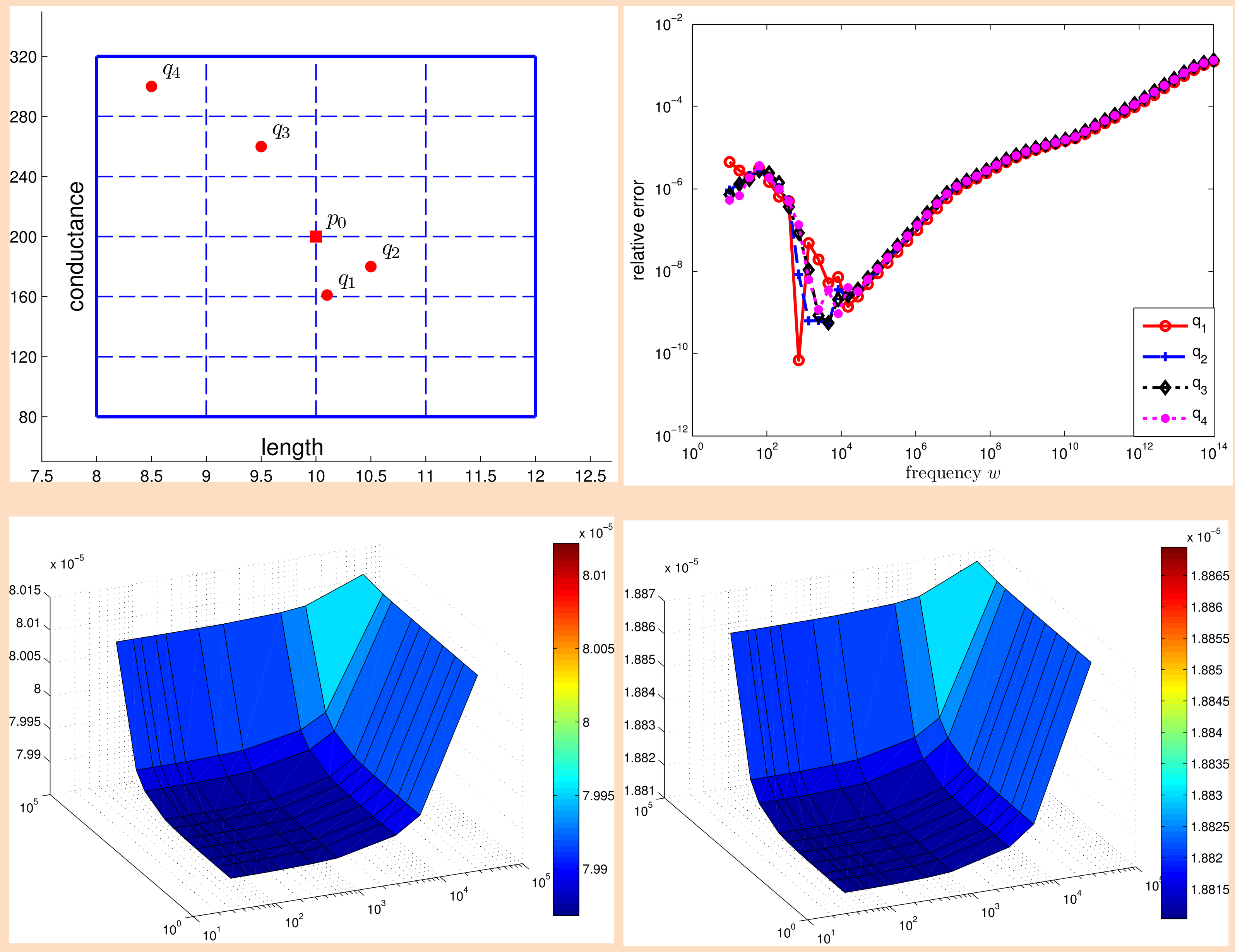
Offline-online decomposition

- In [Asallem Farhat '08], the online stage is not in real time.
- Combination of affine dependence on parameters

$$E(p) = \sum_{i=1}^{\Phi_E} f_i^E(p) E_i, \quad A(p) = \sum_{i=1}^{\Phi_A} f_i^A(p) A_i,$$
$$B(p) = \sum_{i=1}^{\Phi_B} f_i^B(p) B_i, \quad C(p) = \sum_{i=1}^{\Phi_C} f_i^C(p) C_i,$$

analysis of sums of SVDs and offline-online decomposition, the computational complexity of the online stage is $O(r^3)$, where r is the reduced order [Son '12].

Numerical Examples



- **Information on the reduction**

	ori./red. order	♯ input	♯ output	♯ parameters
Circuit:	6013/49	4	4	2
Microthruster:	4257/40	1	1	3

- **Explanations of the result**
  - Upper left figure: the grid in parameter domain used for interpolation in the circuit example.
  - Upper right figure: relative error in frequency domain at four points q_1, q_2, q_3, q_4.
  - Lower figuges: Relative errors w.r.t. $\mathcal{H}_\infty$ norm for the microthruster example in the parameter domain with $h_{top} = 200$, reduced order 30 (left) and 40 (right). We use only four grid points at the four vertices of the parameter domain.
- **Computation acceleration**

Time (counted in second) consumed to compute reduced order models in the microthruster example at 99 points in parameter domain and at different reduced orders. They are computed by two ways, the proposed method (With off-on decomp.) and the method proposed in [Amsallem/Farhat '08] (Without off-on decomp.)

Reduced order	10	20	30	40
With off-on decomp.	0.0674	0.1982	0.4562	0.8372
Without off-on decomp.	1.0480	3.0708	5.8994	7.5586
Acceleration factor	15.5415	15.4934	12.9309	9.0287

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