Max-Planck-Institut für
Dynamik komplexer technischer Systeme
Max Planck Institute for Dynamics of Complex Technical Systems

MAX–PLANCK–INSTITUT
FÜR DYNAMIK KOMPLEXER
TECHNISCHER SYSTEME
MAGDEBURG

Thesis Topic

Research Group
"Computational Methods in Systems and Control Theory"

Dipl.-Math. Matthias Voigt

Computational Methods in Systems and Control Theory

Phone: +49 391 6110 464
Fax: +49 391 6110 500

Email:
voigtm@mpi-magdeburg.mpg.de

www:
http://www.mpi-magdeburg.mpg.de/mpcsc/voigtm/

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Title

"Numerical Computation of Generalized Structured Pseudospectra"

Job Description

In many fields of pure and applied mathematics, the behavior of matrices and operators is analyzed by pseudospectra. For a given matrix $A \in \mathbb{C}^{n\times n}$, the ε-pseudospectrum is defined by

$$\Lambda_\varepsilon(A) := \left\{ z \in \mathbb{C} : z \in \Lambda(A+\Delta) \text{ for some } \Delta \in \mathbb{C}^{n\times n} \text{ with } \|\Delta\|_2 < \varepsilon \right\}. \quad (1)$$

A set of such pseudospectra is depicted in Figure 1.

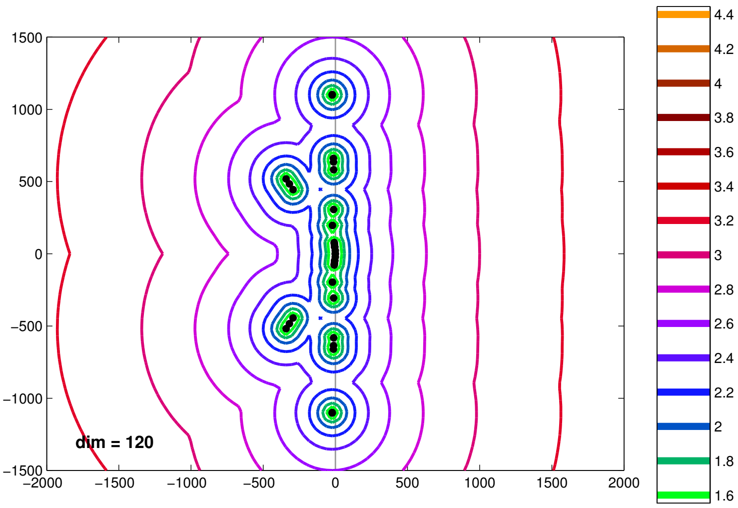

Figure 1: pseudospectra of a matrix

In some applications in systems and control theory one considers a generalization of (1), namely the structured ε-pseudospectrum for the matrix pencil $\lambda E - A \in \mathbb{C}^{n\times n}$ with respect to $B \in \mathbb{C}^{n\times m}$ and $C \in \mathbb{C}^{p\times n}$, given by

$$\Lambda_\varepsilon(E,A,B,C) \\ := \left\{ z \in \mathbb{C} : z \in \Lambda_f(E, A+B\Delta C) \text{ for some } \Delta \in \mathbb{C}^{m\times p} \text{ with } \|\Delta\|_2 < \varepsilon \right\},$$

where $\Lambda_f(M,N)$ denotes the finite spectrum of the matrix pencil $\lambda M - N$.

Sandtorstraße 1
39106 Magdeburg
Phone: +49 (0)391 6110 450
Fax: +49 (0)391 6110 453
E-Mail: holzmann@mpi-magdeburg.mpg.de
http://www.mpi-magdeburg.mpg.de

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It can be shown that

$$\Lambda_\varepsilon(E,A,B,C) = \Lambda_f(E,A) \cup \left\{ z \in \mathbb{C} : \sigma_{\max}\left(C(zE-A)^{-1}B\right) > 1/\varepsilon \right\},$$

where $\sigma_{\max}(M)$ denotes the maximum singular value of M.

Then the obvious algorithm for computing and plotting structured pseudospectra consists of evaluating $\sigma_{\max}\left(C(zE-A)^{-1}B\right)$ on a grid and passing the data to a contour plotter. However, for larger matrices and finer grids, the computation becomes prohibitively expensive.

For the matrix case, possibilities for an acceleration of the computation have been proposed in [1]. The task of this thesis is to generalize the results obtained in [1] to the structured pseudospectrum case. A MATLAB program for generating pseudospectrum plots as in Figure 1 should be written. Finally, the computational complexity and the quality of the results should be compared with the simple algorithm.

References

[1] Trefethen, L. N.: *Computation of pseudospectra*, Acta Numerica, Cambridge University Press, pp. 247–295, 1999.

[2] Trefethen, L. N., Embree, M.: *Spectra and Pseudospectra: The Behavior of Nonnormal Matrices and Operators*, Princeton University Press, Princeton, 2009.

[3] Wright, T. G.: *EigTool.* `http://www.comlab.ox.ac.uk/pseudospectra/eigtool/`, 2002.

[4] Benner, P., Voigt, M.: *Structured Pseudospectral Methods for $\mathcal{H}_\infty$-Norm Computation of Large-Scale Descriptor Systems*, in preparation.

Job Requirements
Recommended: Numerical Analysis, Numerical Linear Algebra (Eigenvalue Problems),
Desirable: Systems and Control Theory.

Degree
Bachelor