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Modeling Glyphosate Aerial Spray Drift at the Ecuador-Colombia Border

Peter Benner* Hermann Mena† René Schneider‡

Abstract

We propose a mathematical model for the Glyphosate aerial spray drift at the Ecuador-Colombia border. Glyphosate is one of the herbicides used by the Colombian government to spray coca fields close to the Ecuadorian border. The model considers the particular procedures of the sprays at the border as well as the topography and weather conditions of three zones of interest at the border. Two dimensional simulations are shown. The lack of reliable information constrains the accuracy of the model. However, the results presented in this work can be used as a starting point for more accurate models of the phenomena.

Keywords: Ecuador-Colombia border, glyphosate, spray model, drift, aerial application, convection-diffusion equation, numerical simulation.

1 Introduction

Glyphosate is one of the herbicides used by the Colombian government to spray coca fields close to the Ecuadorian border. The sprays have taken place for a number of years and have been more frequent after 2000, when Plan Colombia started.

Spray drifts into Ecuadorian territory became a big issue for people living close to the border. Their negative impact on health and agriculture have been observed and confirmed by intensive studies, e.g., [1]. Hence, in 2005 Ecuador and Colombia signed an agreement to stop the sprays in a 10 km corridor along the border. However, measurements on Ecuadorian territory indicated that significant amounts of Glyphosate spray have still drifted into Ecuador. The sprays stopped in 2007 and a trial in the International Court of Justice started. In September 2013 the case was settled with an agreement that "sets out operational parameters for Colombia's spraying programme, records the agreement of the two Governments to ongoing exchanges of information in that regard, and establishes a dispute settlement mechanism" [21]. In the settlement Colombia also agreed to pay 15 million US dollars to Ecuador [22].

The herbicide/pesticide aerial spray drift is a big concern in agricultural communities. Most studies of spray drift so far have focused on the extent of near-field drift under varying meteorological conditions and application methods [11, 12, 15]. Application procedures and general guidelines have been proposed in the context of agriculture, in order to maximize the effectiveness of plant protection products and minimize risks to public health and the environment, see e.g. in [7]. All the models in the literature assume that these procedures and guidelines are followed. However, for the sprays at the Ecuador-Colombia border some of these guidelines either cannot be followed, e.g. the maximum aircraft spray height due to the topography of zone, or they were not followed, e.g., the droplet size, see [1] and references therein. These result in demands for a new model that considers the particular spray procedures at the border and deals with technical difficulties, like the size of the spray zones and the accuracy required.

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In this paper we propose a mathematical model and perform two-dimensional numerical simulations of the spray drift in specific zones at the border. Even though the Ecuador-Colombia case was settled for now, similar issues may arise elsewhere. Our model is not specific to the herbicide, it could be applicable even in very different situations.

The paper is organized as follows. First, we briefly study the physical phenomena of the aerial spray drift in two zones: close to the nozzle and distant from the nozzle. Then, we review the existing models that have been proposed in the literature and state the aerial spray guidelines. After that, we describe the particular procedures of the sprays at the Ecuador-Colombia border. Then, we propose our mathematical model and show some preliminary numerical simulations using data similar to the three zones of interest. Finally, some discussion of the results and conclusions are stated.

2 Physical phenomena

The aerial spray can be seen as two-phase fluid flow, where liquid droplets are released into an air flow. In order to model this situation it is necessary to determine both the air flow in the system and the spray movement in the prevailing air flow. The spraying process can be divided into two zones: close to the nozzle, where droplet movement is influenced by the sprayer and at distance from the sprayer where droplet movement is controlled by prevailing meteorological conditions, see Figure 1.

Models of droplet movement in the near nozzle region are often ballistic or particle trajectory models. Close to the spray nozzle the spray is relatively dense and the droplets can influence the local air turbulence [6]. The fact that droplets are being propelled from the nozzle in a certain direction causes surrounding air to be entrained into the spray plume [16]. The combination of the high droplet concentration, initial spray sheet and entrained air can provide a blockage to cross flowing air, resulting in regions of low and high air pressure, leading to the creation of spray induced vortices [28, 30]. The spray vehicle, e.g., aircraft, and spray structures, e.g., booms, can also create additional turbulence in the region where the spray is being produced. The modeling near to the nozzle has been studied extensively [5, 19, 20, 29]. The main difference between these models relate to how air flow is characterized in the near nozzle region.

On the other hand, once a droplet moves far enough from the spray nozzle it will move entirely under the influence of the prevailing meteorological conditions. At this stage the spray concentration in the air is low so the influence of the droplets on the local air turbulence is negligible [6]. The main purpose of models of droplet movement at distance from the spray nozzle is to determine the amount of spray drift moving away from treatment areas. For our problem we are particularly interested in modeling this phenomenon. Two main approaches that have been used for these models are Gaussian diffusion theory and random walk. We will briefly review these approaches in the next section.

3 Review of existing models

A number of models have been developed to predict the drift and deposition from aerial spray applications [2, 3, 9, 10, 13, 14, 18, 33, 37, 42]. These aerial spray models fall into two general categories: empirical and mechanistic. The empirical models do not take into account any physical basis for the spray drift and are generally applicable only to situations very similar to those for which they were developed, e.g., [18, 33].

Mechanistic models are based on Gaussian dispersion equations and particle tracking models (Lagrangian particle trajectory) [35]. Gaussian modeling [2, 9, 10, 37] is a classical approach used in atmospheric dispersion modeling of releases from tall stacks and line, area, and volume sources and is well suited for modeling moderately long-range drift (0.5 km) and simulating the effects of atmospheric stability. However, the Gaussian approach does not provide much resolution in the representation of equipment and near-field dynamics in the flow field near the aircraft. Lagrangian models [3, 13, 42], on the other hand, track a cohort of droplets in a given drop size category and overlay a random component on the movement of the droplets to account for atmospheric turbulence. The Lagrangian approach lends itself to detailed modeling of the effects of application equipment on spray dispersal and thus, as an approach, most effectively meets the needs for a regulatory assessment tool that can be used to evaluate the mitigating effects of alternative equipment uses and near-field buffer zones. Most of the methods do not provide finely

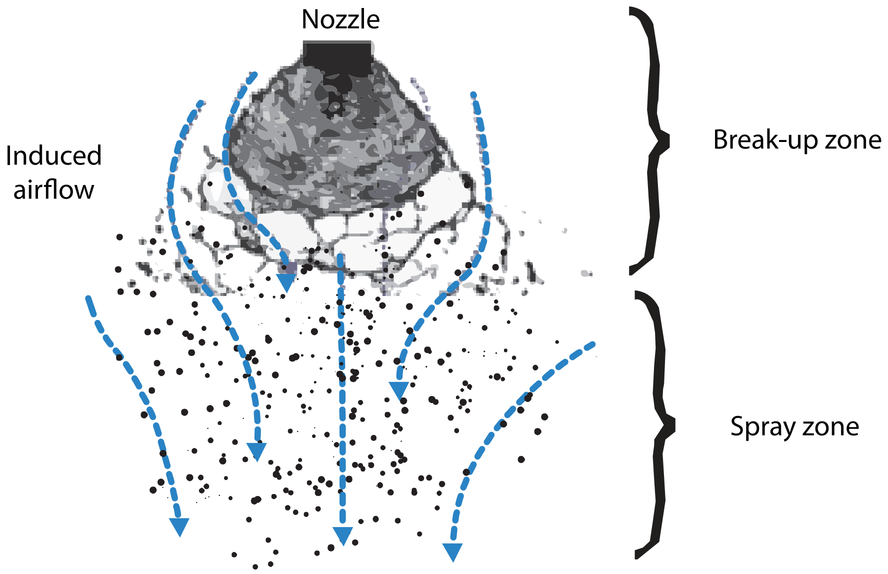

Figure 1: Diagram of the airflow near and distant to the nozzle [7, Section 2.1]

time-resolved data but are instead integrated over relatively long time intervals (sampling periods). Long sampling periods (on the order of hours) compared to the relatively short spray event (an hour for a 34.4 ha crop circle) make it difficult to analyze the drift evolution.

Gaussian plume models make assumptions about the shape of a plume and the distribution of material within a plume [38]. A spray nozzle moving along a field (either on a boom spray or agricultural aircraft) is assumed to produce an instantaneous line source of droplets as the time taken to release the spray is short compared to the time scale of the atmospheric turbulence that affects the spray dispersal. A cloud of droplets released from such a line source is subject to the meteorological conditions. A scheme of a Gaussian model is shown in Figure 2.

In random walk models the trajectory of each droplet is followed as it moves through the atmosphere [17]. A meaningful estimate of dispersal statistics can be obtained by following a large number of trajectories. The trajectory of each fluid particle is divided into a large number of small discrete time steps of constant duration, during which the velocity components of the particle are kept constant. Models based on a random-walk approach have been shown to have good agreement with experimental data close to and distant from the source [17, 41]. A random walk model generally ignores near nozzle effects, however, by tracking the trajectory of droplets in discrete time steps it is possible to account for some near nozzle influences.

Some state-of-the-art models are the US Environmental Protection Agency (EPA) Fugitive Dust Model (FDM) [43] and AgDRIFT [36], which is a Lagrangian type model developed by the Spray Drift Task Force (a consortium of agricultural chemical companies) in collaboration with the US EPA. AgDRIFT is used as a regulatory tool to efficiently fulfill EPA's spray drift data requirements for pesticide registration in the United States. However, the AgDRIFT model interface is not amenable to modeling an actual spray event where there is changing meteorology and a moving source.

The FDM model offers flexibility in defining changing meteorological, source, and size distribution input parameters. Furthermore, FDM output can provide time-resolved concentration measures at user-defined receptor locations for a particular spray event. The main drawback of using FDM was its inability to model evaporation. A calibrated model used for retrospective reconstruction of exposure is proposed in [39].

Finally, we point out that modeling is essential for estimating the spread, deposition concentration, and time evolution of spray drift. A model of the ground deposition component of drift is proposed in [39]. The gas phase concentrations due to post-spray volatilization of the pesticide from wetted fields are discussed

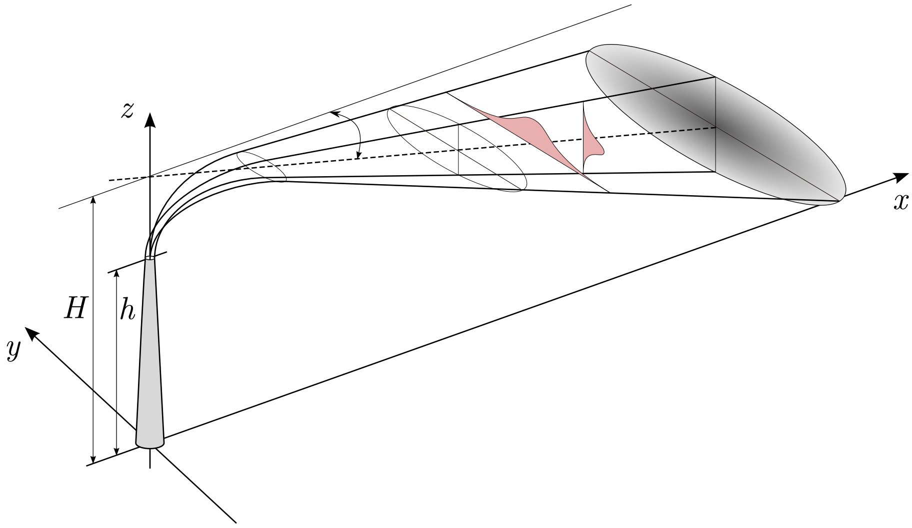

Figure 2: Scheme of a Gaussian model (h - actual stack height, H - effective stack height due to buoyancy effects)

Place	Area (km^2)	Longitude	Latitude	Wind (km/h)
El Conejo	8.8 x 10	0.23	-76.90	7
San Marcelino	12.9 x 10	0.24	-76.76	6
Chanangue	16.0 x 10	0.23	-76.60	5

Table 1: Area, location and average wind for the zones of interest

in [31].

In order to make accurate predictions these models require inputs representing: data of the aircraft, aircraft flight conditions, data of the nozzles, the drop size distributions, the pesticide/herbicide composition and properties, meteorological conditions. Moreover, some of these data values have to be in a certain range, e.g. the speed of the aircraft, in order to fulfill the assumptions of the model required for accurate results. In the case of AgDRIFT (perhaps the most advanced model) this model is meant to simulate the near-field spray drift [36].

4 Aerial sprays at Ecuador-Colombia border

Most of the models, e.g., the AgDrift [36], require at least input data representing: a) aircraft flight conditions, b) the nozzles, c) the droplet size distribution, d) the spray material properties, e) the meteorology conditions. For the sprays at the Ecuador-Colombia border: a), b) and c) are not known. Moreover, d) and e) are difficult to estimate due to the fact that the exact composition of the herbicide is not known and on the other hand there are no weather stations near to the zones of interest, i.e., the areas where the sprays took place. These regions were chosen in cooperation with an interdisciplinary team of biologists, engineers, and geophysicists investigating the effects on human beings, animals and the grounds. In Figure 3 these zones as well as the direction of the average wind are visualized. Table 4 contains the location of the zones.

The following are some international guidelines for aerial sprays [27, 40]: verify the direction of the wind, no application within 46 m of an unprotected person, use largest droplet size (the minimum recommended size is 500 μm), spray when wind speeds are between 1.3 and 4.5 m/s, avoid spraying in low

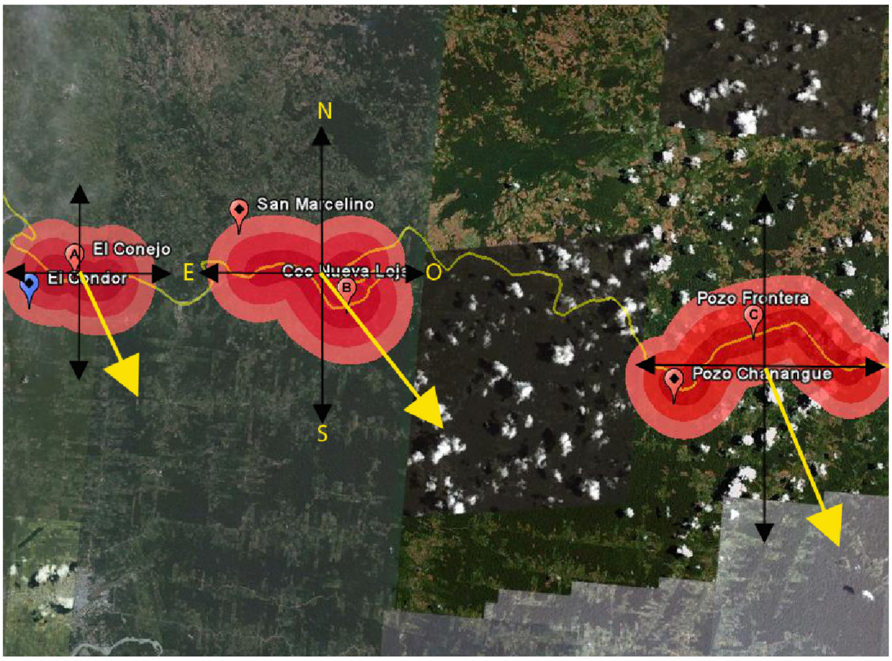

Figure 3: Zones of interest at Ecuador-Colombia border and direction of average wind

humidity and high temperature conditions, do not spray during temperature inversions, maximum spray height of the aircraft is 25 m.

In the particular case of the sprays at the Ecuador-Colombia border, the average droplet size was 150 μm, the spray height of the aircrafts was up to 80 m [1], in some zones there are low humidity and high temperature conditions.

Taking all of these into account we conclude a model for the Glyphosate aerial spray drift at the Ecuador-Colombia border has to fit the following criteria: small droplet size, diffusion and transport are the dominant phenomena, simulation domains are considerably large compared to the size of droplet sources, sprays take place at a height higher than the recommended maximum (25 m), which implies more evaporation and more drift, many input parameters are unknown. In the following section we propose a mathematical model governed by partial differential equations, since the long term goal is to model the phenomena as a tracking optimal control problem in order to assess whether the agreement signed by Ecuador and Colombia to stop the sprays in a 10 km corridor along the border was respected. This, however, will be left to future work.

5 Mathematical model

We propose a model governed by partial differential equations. However, a full-physics multi-phase Navier-Stokes approach is currently not feasible for the relevant combinations of very small source length-scales (100 μm), domain sizes (10 km) and time-scales. Instead we propose a simplified model, to characterize qualitatively and quantitatively the spray drift.

First we assume that the spray plume released by the aircraft is composed of a range of droplet sizes, whose distribution is dependent on actual operation conditions, e.g. break-up of droplets due to turbulence. The larger droplets will fall down within a relatively short distance of the release point. These are not relevant to the presumed phenomenon of significant amounts of herbicide drifting 10 km or more.

Very small droplets however, have a very low mass in relation to their surface area. Thus, they do not just fall down, but behave more like mist droplets, suspended in the surrounding air. Their movement is primarily dictated by the motion of the air they are suspended in. These droplets can travel long distances, transported by the wind.

Of course there may be a whole range of droplet sizes between these extremes, where the smaller droplets quickly attain a constant velocity of fall, the terminal velocity, [8, Section 12.6], if the surrounding air is at rest. As we will see, the terminal velocity is droplet size dependent. It is defined as the velocity for which the drag force equals the gravitational force on the droplets (buoyancy can be neglected here). The drag force on a moving object in air is in general

$$F_D = \frac{1}{2}\rho_{\text{air}}\, c_D\, A\, v^2,$$

where c_D is the drag coefficient, v the velocity of the droplet relative to the air, $\rho_{\text{air}} \approx 1.23\ \text{kg/m}^3$ the density of air and A the area of the cross-section of the droplet in direction of its movement. The drag coefficient is dependent on the Reynolds-Number

$$\text{Re} = \frac{\rho_{\text{air}}\, v\, d}{\mu}, \tag{1}$$

a dimension-less quantity describing the flow regime, which is defined by velocity v, droplet diameter d, density ρ_{air} and viscosity $\mu \approx 1.78 \cdot 10^{-5}\,\text{kg}\,\text{m}^{-1}\,\text{s}^{-1}$. We assume Stokes-flow ($\text{Re} \leq 0.2$, so $c_D = 24/\text{Re}$) and spherical droplets ($A = \pi\, d^2/4$), thus leading to drag force

$$F_D = 3\pi\,\mu\, d\, v$$

and gravitational force

$$F_g = \frac{\pi\, d^3}{6}\rho_{\text{herb}}\, g$$

with gravitational acceleration $g \approx 9.81\,\text{m}\,\text{s}^{-2}$ and herbicide density $\rho_{\text{herb}} \approx \rho_{\text{water}} \approx 1000\ \text{kg/m}^3$. Setting $F_D = F_g$ and solving for v yields the terminal velocity

$$v = d^2\,\frac{\rho_{\text{herb}}\, g}{18\,\mu}. \tag{2}$$

So the terminal velocity is $\mathcal{O}(d^2)$. If the surrounding air is not at rest but moving with constant (wind) velocity, the velocity vector of the air adds to the terminal velocity vector.

To put this into perspective, we consider three example cases:

1. For the assumed average droplet size $d = 150\,\mu\text{m}$, (2) leads to $v = 0.689\,\text{m/s}$, which implies $\text{Re} = 7.14 > 0.2$. Thus for this size the Stokes-flow is not an appropriate model, implying that the actual c_D will be smaller, and thus the actual terminal velocity be larger. However, this allows a crude upper estimate on the average distance these droplets drift before they hit ground, when they are released at an altitude of 80 m into air at wind-speed 7 km/h, i.e. they will drift less than 226 m.

2. Substituting (2) into (1) we see that Stokes-flow, $\text{Re} \leq 0.2$, will be a valid model for $d \leq 45.5\,\mu\text{m}$. For $d = 45.5\,\mu\text{m}$ we get a terminal velocity of $0.0635\,\text{m/s}$, implying that under the conditions as above droplets of this size may drift up to 2.45 km.

3. For $d = 5\,\mu\text{m}$ we get a terminal velocity of $7.7 \cdot 10^{-4}\ \text{m/s}$, so these droplets could drift up to 203 km.

The last example illustrates that the droplets being suspended in the surrounding air is an appropriate model for small d, say for $d \leq 5\,\mu\text{m}$.

As we are not primarily interested in the paths of individual droplets, we move on to averaged concentrations of droplets in the air, and even further, to concentration of the herbicide in the air. This consideration of the herbicide concentration has the advantage that evaporation of the herbicide is implicitly also contained in the model, since it does not distinguish between herbicide dissolved in droplets suspended in air and herbicide dissolved in air.

The dynamics of the concentration over long timescales are mainly described by two effects:

- transport (convection) due to the velocity of the surrounding air and the (very slow) fall of the small droplets and
- diffusion, i.e. local smoothing of concentration contrasts due to small scale random particle motion.

This leads us to the convection-diffusion equation

$$\partial_t c - \nabla.(a\nabla c) + b^T \nabla c = f \qquad \text{in } \Omega \times (0,T) \tag{3}$$

where

- $c = c(\mathbf{x}, t)$ is the (unknown) concentration,
- $a = \begin{bmatrix} k_x & 0 & 0 \\ 0 & k_y & 0 \\ 0 & 0 & k_z \end{bmatrix}$ is the diffusion coefficient, $k_x > 0$, $k_y > 0$, $k_z > 0$,
- b the wind speed vector and
- $f = f(\mathbf{x}, t)$ the source term, the concentration release rate by the moving source (airplane).

For the wind only the average direction and speed are known, whereas the above considerations assume accurate knowledge of the whole air velocity field over space and time. Temporal and spatial variations of this velocity field are described by atmospheric turbulence, which acts additionally to disperse the concentration field. This dispersion can be modeled by an additional diffusion [24]. Usually the dispersion (due to turbulence) strongly dominates the diffusion (due to random particle motion). Thus we use diffusion coefficients k_x, k_y and k_z purely on the basis of the dispersion model [24, Table 1]. I.e. for an average wind speed of $3\,\mathrm{m/s}$ we use Pasquill stability class C, "Slightly unstable", [4, Table 10.2], leading to $k_x = k_y = 30\,\mathrm{m^2/s}$ and $k_z = 6\,\mathrm{m^2/s}$.

For simplicity and lack of more accurate data the domain is chosen to be a cuboid $\Omega := (0, \ell_x) \times (0, \ell_y) \times (0, \ell_z)$.

In order to simulate a single pass of the spraying airplane along a fixed straight path in otherwise herbicide-clean air, we choose zero initial concentration, homogeneous Dirichlet boundary conditions at the inflow boundaries, natural boundary conditions everywhere else,

$$\begin{aligned} c(\mathbf{x}, 0) &= 0 && \forall \mathbf{x} \in \Omega \\ c(\mathbf{x}, t) &= 0 && \text{on } \Gamma_{\text{in}}, \\ (a\nabla c)^T n &= 0 && \text{on } \Gamma_{\text{N}} := \partial\Omega \setminus \Gamma_{\text{in}}. \end{aligned}$$

The natural boundary conditions can be interpreted as *no diffusion through this boundary*, leaving only transport to remove herbicide from the domain.

The source f is chosen to be a Gaussian with a moving center and standard deviation of $\sigma = 10\,\mathrm{m}$ (approximate length scale of the airplane wing span),

$$f(\mathbf{x}, t) = c_f \exp\left(\frac{-||\mathbf{x} - (\mathbf{x}_0 + t\mathbf{v})||_2^2}{2\sigma^2} \right),$$

with position $\mathbf{x}_0 \in \mathbb{R}^3$ where the spraying starts at time $t = 0$ and airplane velocity $\mathbf{v} \in \mathbb{R}^3$. The intensity $c_f > 0$ of the source term has to be chosen to match measurements. If there is nothing known on this source intensity, one can just choose $c_f = 1$ and study the development of the concentration relative to this source intensity, since the PDE is linear, and hence the PDE solution is linear in this source parameter.

The deposition rate $d(\mathbf{x}, t)$ of the herbicide to the ground, i.e. the amount of herbicide that leaves the domain through the bottom boundary, is given by

$$d(\mathbf{x}, t) = (b^T n)\, c,$$

where n is the outward normal of the boundary, i.e. so far the boundary condition on the bottom models only deposition of the herbicide on the ground by the convective term. Other possibilities would just involve different boundary conditions. Since no data is available to support a more elaborate deposition model, this is not considered further in this present work.

6 Numerical simulation

The PDE (3) which forms the mathematical model for the spray drift comprises an instationary convection diffusion equation. Our preliminary numerical simulation treats the problem on two dimensional bounded domains $\Omega \subset \mathbb{R}^2$, in order to study properties of the model and feasibility of computations on three dimensional domains with realistic parameters of Section 4.

We use the method of lines approach to separate the discretisation in space and in time. The space discretisation is performed first. It uses P_2 finite elements on meshes of triangles. This results in an ODE system with typically a large number of states and equations. To solve these resulting ODE systems we use the third order Rosenbrock Method ROS3P [25, 26], with the embedded lower order scheme for step size control. This has been incorporated in the finite element software package `FEINS` [34].

For convection diffusion equations un-physical oscillations may appear in solutions computed with the Galerkin finite element method if convection dominates diffusion. These oscillations result in under and overshoots for the concentration. Especially the undershoots below zero are not compatible with the concept of concentration, thus they complicate interpretation of computational results. For time dependent problems this phenomenon and attempts to overcome it are discussed in [23]. A very popular approach is to use the Streamline Upwind Petrov Galerkin (SUPG) method, where certain terms are added to the discrete version of the PDE in weak formulation in order to improve stability, i.e. reduce the oscillations. While this approach is fairly simple to implement and is often successful in reducing the oscillations, it is usually not sufficient to remove them completely. It is widely acknowledged that the most robust numerical schemes in this respect combine special refined meshes with a stabilised method, e.g. [32]. The stabilisation is not in general necessary though, if it is possible to use meshes with sufficient refinement in critical areas of the domain.

Due to the modelled dissipation, the diffusive terms in (3) are sufficiently large, therefore at least on reasonably fine meshes the convection term is not dominant in our case. Thus it is possible to work without stabilisation. Indeed our computational results without stabilisation in the first example domain below show undershoots to only $-6.9 \cdot 10^{-4}$ which are confined to very small regions of the domain. Experiments with SUPG stabilisation, even with different choices of the stabilisation parameter as suggested in [23], did not lead to substantial reductions in these undershoots.

In order to asses feasibility of the computations and properties of the discretisation, we consider two examples:

1. a small example domain of $\ell_x = 2\,\text{km} \times \ell_y = 2\,\text{km}$ with wind $b = (0.9487, -2.846)^T\,\text{m/s}$, so $||b|| = 3.0\,\text{m/s}$,

2. the applications actual domain of $\ell_x = 16\,\text{km} \times \ell_y = 10\,\text{km}$ with wind $b = (0.9821, -0.9821)^T\,\text{m/s}$ so $||b|| = 1.3889\,\text{m/s} = 5\,\text{km/h}$.

The first of these was used primarily during the development of the software, while the more realistic second example was introduced later in this work, with the aim of further assessing feasibility of computations for the actual domains in 3D.

In both cases the inflow boundary is given by $\Gamma_{\text{in}} := \left\{\mathbf{x} \in \mathbb{R}^2 : \quad x_1 = 0 \text{ or } x_2 = \ell_y\right\}$. The initial airplane position is assumed to be $\mathbf{x}_0 = \left[100\,\text{m}, \quad \ell_y - 100\,\text{m},\right]^T$ and velocity $\mathbf{v} = \left[90\,\text{m/s}, \quad 0\right]^T$.

First example, 2 km $\times$ 2 km: The discretisation on a uniform mesh of $257 \times 257 = 66,049$ nodes showed undershoots to only -0.000101245. However, the resolution even on this relatively fine mesh is quite coarse with respect to the size of the spatial area where the source term is effective, see Figure 6. In order to asses this difficulty, the problem has also been studied using a locally refined mesh which is refined in the area where the source term has direct influence. Figure 7 shows the "hand" refined mesh with $35,453$ nodes and corresponding solution at different times. The disadvantage of the locally refined mesh is that resolution further away from the source area is lower, thus the accuracy is lower once the bulk of the spray concentration leaves the refinement zone. Thus no significant advantage seems to be gained by this "ad-hoc" refinement. Slightly larger undershoots to -0.000690685 occur on this mesh.

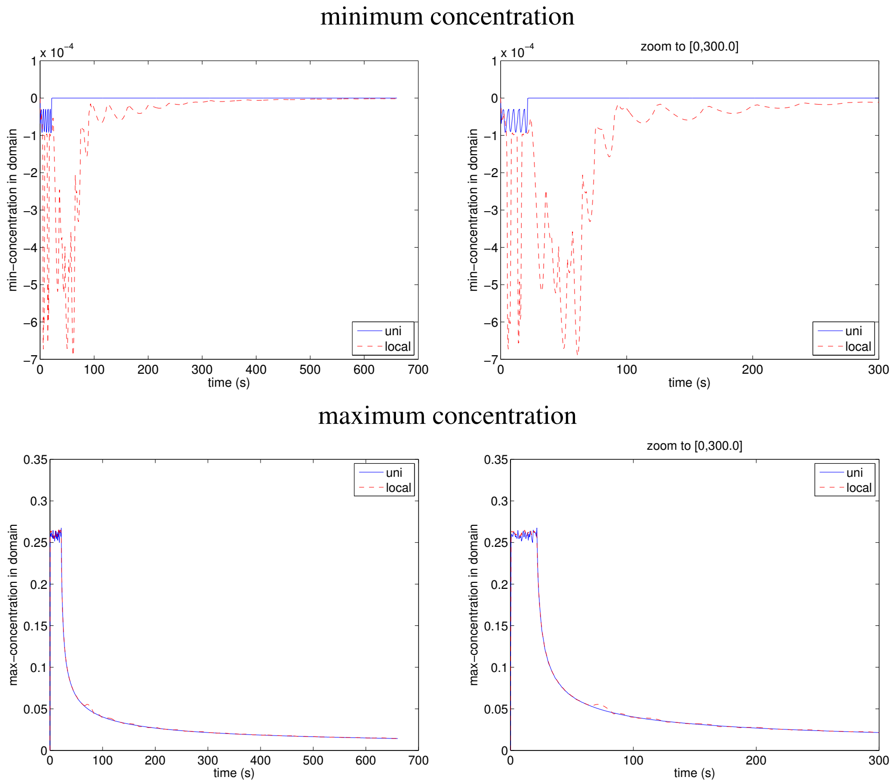

Figure 4: Comparison of locally refined mesh and uniform mesh for the first example in terms of the minimum (top) and maximum (bottom) concentration over time

However, as Figure 4 illustrates by comparing the minimum and maximum concentration over time, the results obtained with those two meshes compare reasonably well, indicating that the solution is reasonably well approximated on both meshes.

Second example, $16\,$km $\times$ $10\,$km: Again a mesh of $257 \times 257 = 66,049$ nodes is used, which now has different stretching in x and y direction, see Figure 8. The larger element size and stronger wind lead to stronger undershoots compared to the first example, to -0.1102. As these strong undershoots only occur locally where the source term is active, we compare this again to locally "hand" refined meshes of $37,904$ nodes (local 1) and to one with $53,081$ nodes (local 2), see Figures 9 and 10.

For this example, minimum and maximum concentrations over time are compared in Figure 5 for the three meshes. It is apparent that the undershoots are most severe on the uniform mesh. Also it appears, the resolution of the source term on this mesh is too low, since the maximum concentration is significantly below the value for the "$2\,$km$\times$$2\,$km" example. The locally refined meshes perform significantly better both with respect to source resolution and undershoots. The *local 2* mesh further reduces under and overshoots compared to the *local 1* mesh.

None the less, the spatial resolution which is attainable with reasonable computational resources appears too low for the realistic problem parameters even in these two dimensional test cases. So a more elaborate local refinement technique, e.g. along the lines of [25], is advisable for this problem, especially for computations in 3D. This may be studied in follow up works.

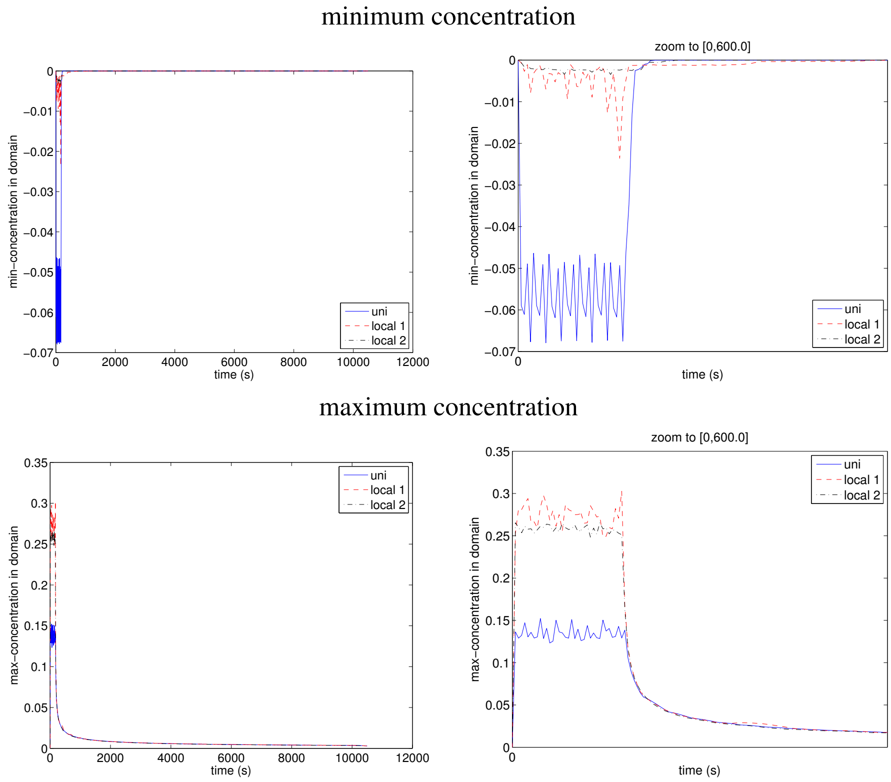

Figure 5: Comparison of locally refined mesh and uniform mesh for the second example in terms of the minimum (top) and maximum (bottom) concentration over time

7 Conclusions

We proposed a mathematical model for the Glyphosate aerial spray drift at the Ecuador-Colombia border. The model considers the particular procedures of the sprays at the border and weather conditions of some spray zones at the border. There is vast, diverse and sometimes contradictory, information about the sprays at the Ecuador-Colombia border. Yet, even the main inputs of our model have to be estimated, e.g., the meteorology conditions are only available as averaged values at nearby locations due to the geographical location. The resulting lack of reliable information constrains the accuracy of the model to an extend that the uncertainty in the results is too high to asses whether the agreement signed by Ecuador and Colombia to stop the sprays in a 10 km corridor along the border has been respected. However, the results presented in this work can be used as a starting point for more accurate models. Even though the Ecuador-Colombia case was settled, similar issues may arise elsewhere. Our model is not specific to the herbicide, it could be applicable even in very different situations.

A three dimensional simulation is required to describe in a better way the spray drifts. Moreover, modeling the phenomena as a tracking optimal control problem might allow better estimates of some of the parameters, thus getting closer to analyze whether the agreement signed by the two countries was respected. This will be left to future work.

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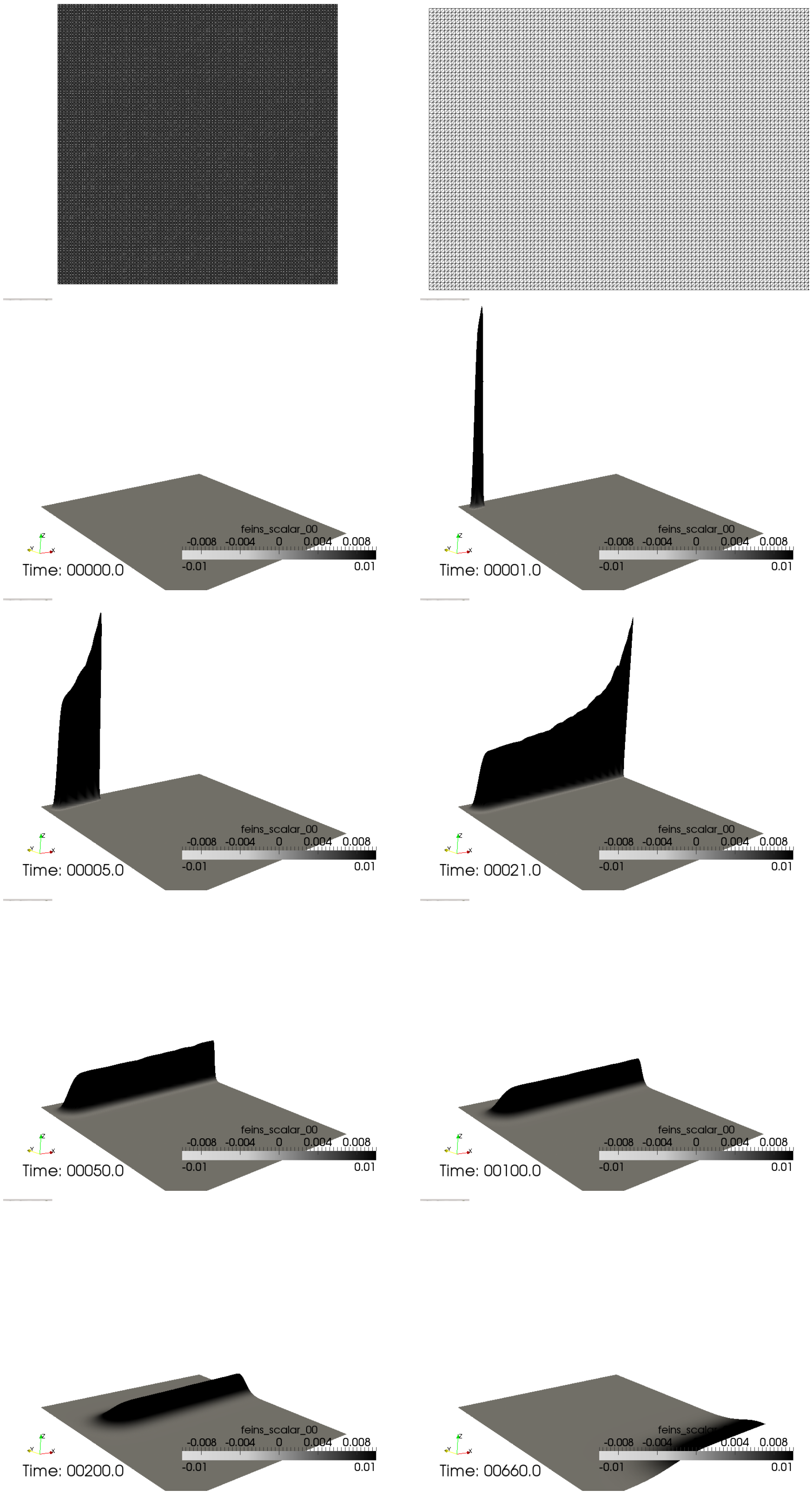

Figure 6: Spray drift simulation on 2 km $\times$ 2 km domain, uniform mesh, $257 \times 257 = 66,049$ nodes, mesh (top-left), zoom of the mesh (top-right) and solution at different times (below)

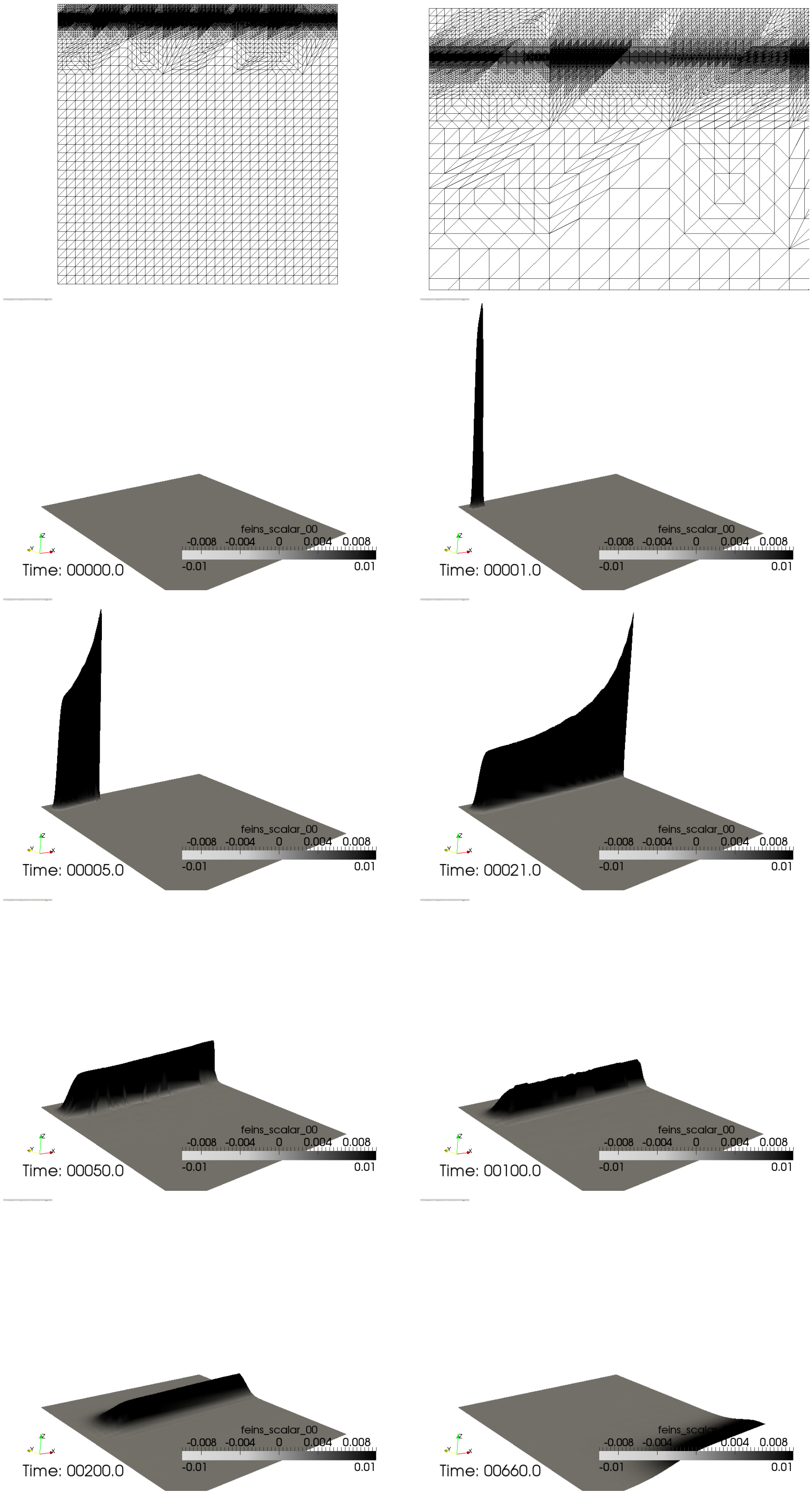

Figure 7: Spray drift simulation on 2 km × 2 km domain, mesh locally refined "by hand", 35,453 nodes, mesh (top-left), zoom of the mesh (top-right) and solution at different times (below)

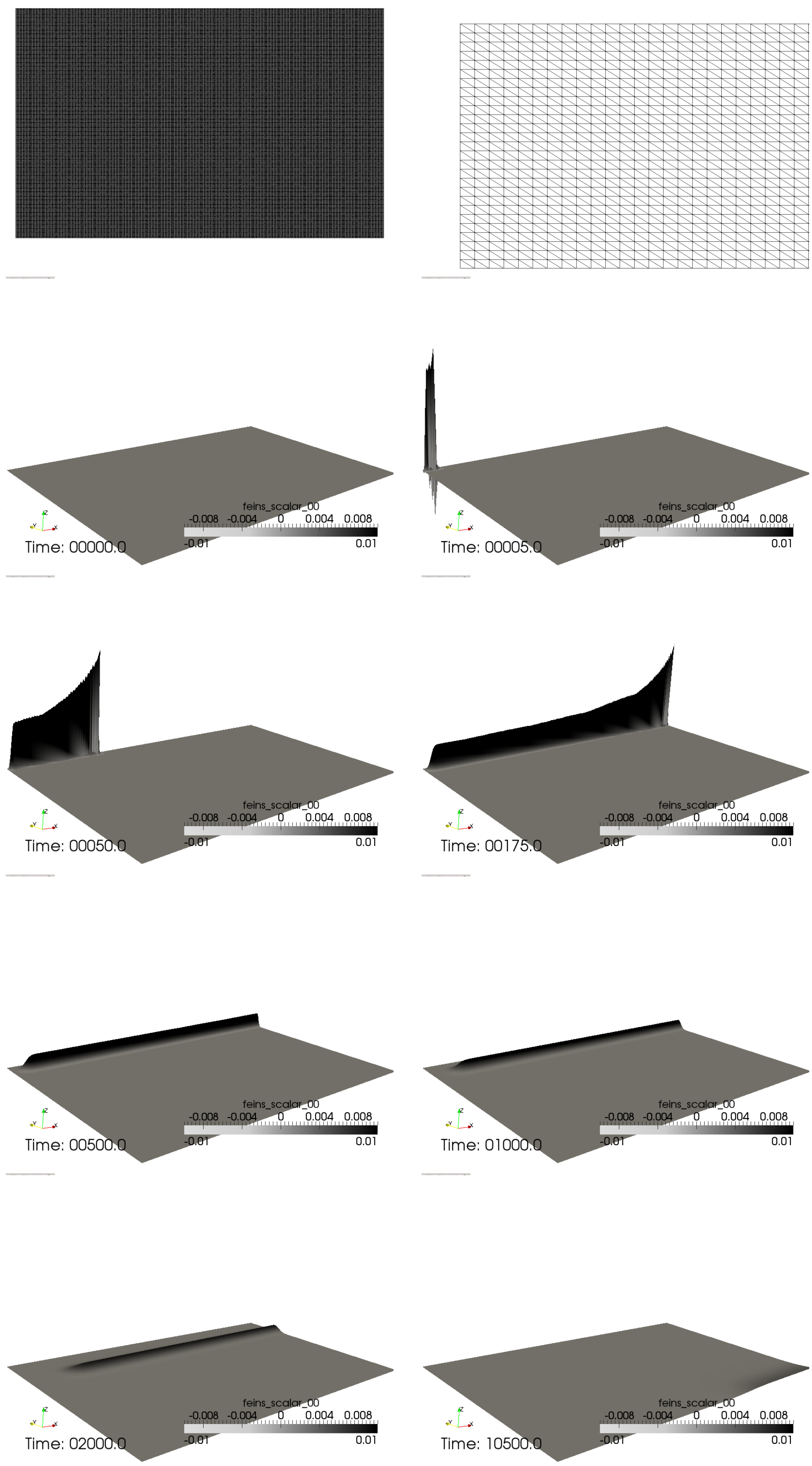

Figure 8: Spray drift simulation on $16\,\mathrm{km} \times 10\,\mathrm{km}$ domain, uniform mesh, $257 \times 257 = 66,049$ nodes, mesh (top-left), zoom of the mesh (top-right) and solution at different times (below)

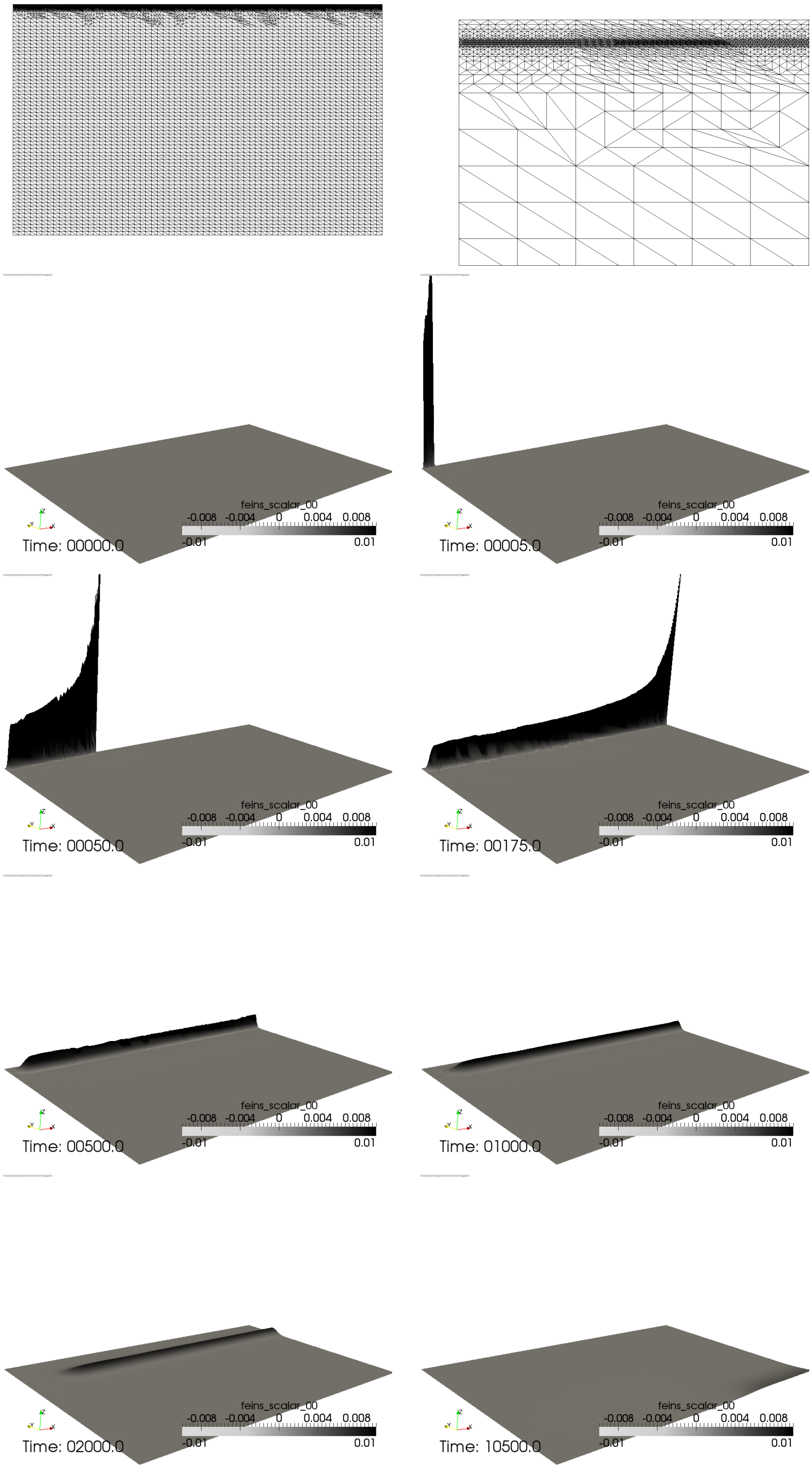

Figure 9: Spray drift simulation on $16\,\text{km} \times 10\,\text{km}$ domain, locally refined mesh (local 1), $37,904$ nodes, mesh (top-left), zoom of the mesh (top-right) and solution at different times (below)

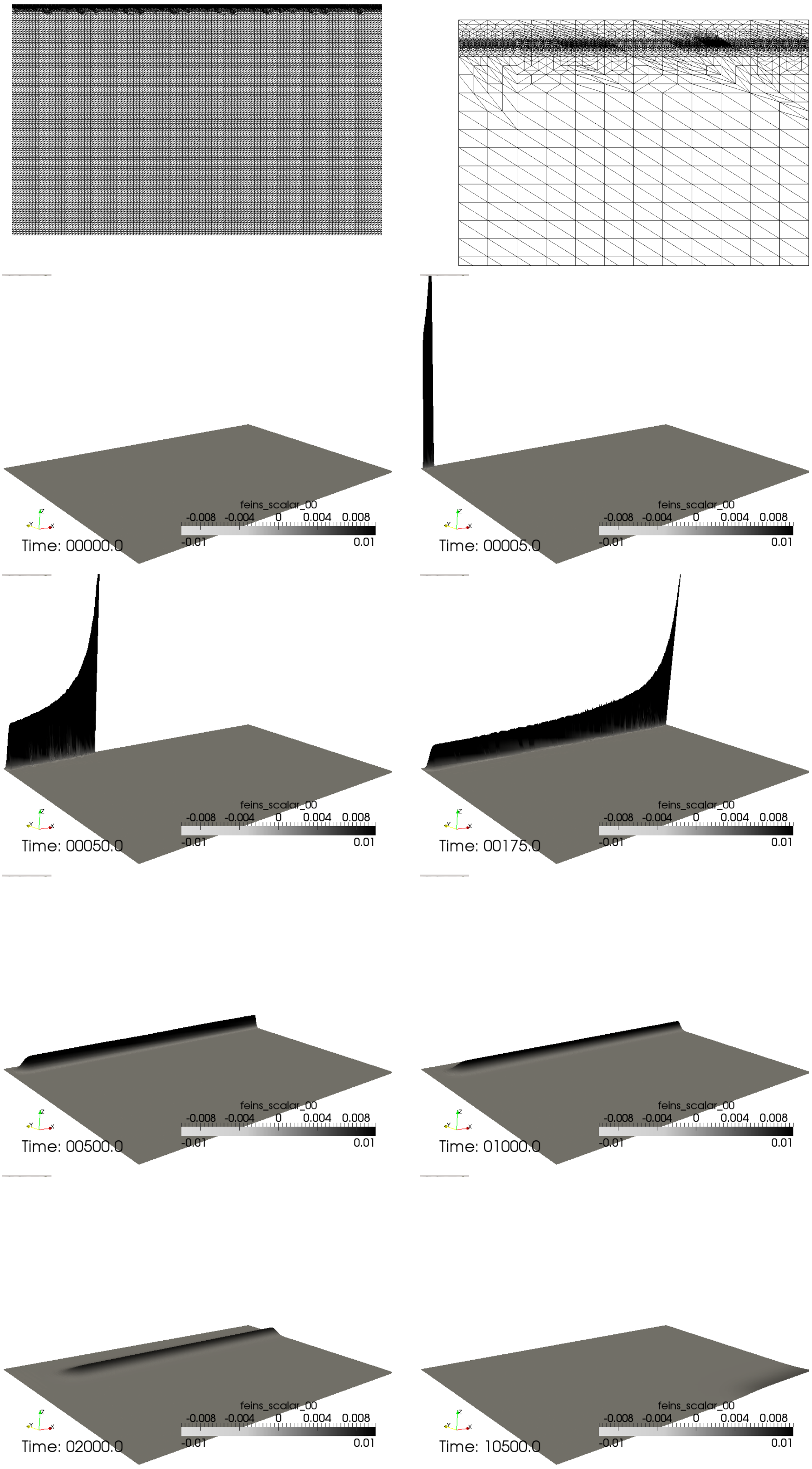

Figure 10: Spray drift simulation on $16\,\text{km} \times 10\,\text{km}$ domain, locally refined mesh (local 2), $53,081$ nodes, mesh (top-left), zoom of the mesh (top-right) and solution at different times (below)